Chapter 3
Solving Linear Equations

What You’ll Learn

- **Lesson 3-1** Translate verbal sentences into equations and equations into verbal sentences.
- **Lessons 3-2 through 3-6** Solve equations and proportions.
- **Lesson 3-7** Find percents of change.
- **Lesson 3-8** Solve equations for given variables.
- **Lesson 3-9** Solve mixture and uniform motion problems.

Why It’s Important

Linear equations can be used to solve problems in every facet of life from planning a garden, to investigating trends in data, to making wise career choices. One of the most frequent uses of linear equations is solving problems involving mixtures or motion. For example, in the National Football League, a quarterback’s passing performance is rated using an equation based on a mixture, or weighted average, of five factors, including passing attempts and completions. You will learn how this rating system works in Lesson 3-9.

Key Vocabulary

- equivalent equations (p. 129)
- identity (p. 150)
- proportion (p. 155)
- percent of change (p. 160)
- mixture problem (p. 171)
Prerequisite Skills  To be successful in this chapter, you’ll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 3.

For Lesson 3-1  Write Mathematical Expressions
Write an algebraic expression for each verbal expression. (For review, see Lesson 1-1.)
1. five greater than half of a number \( t \)
2. the product of seven and \( s \) divided by the product of eight and \( y \)
3. the sum of three times \( a \) and the square of \( b \)
4. \( w \) to the fifth power decreased by 37
5. nine times \( y \) subtracted from 95
6. the quantity of \( r \) plus six divided by twelve

For Lesson 3-4  Use the Order of Operations
Evaluate each expression. (For review, see Lesson 1-2.)
7. \( 3 \cdot 6 - \frac{12}{4} \)
8. \( 5(13 - 7) - 22 \)
9. \( 5(7 - 2) - 3^2 \)
10. \( \frac{2 \cdot 6 - 4}{2} \)
11. \( (25 - 4) \div (2^2 - 1) \)
12. \( 36 \div 4 - 2 + 3 \)
13. \( \frac{19 - 5}{7} + 3 \)
14. \( \frac{1}{4}(24) - \frac{1}{2}(12) \)

For Lesson 3-7  Find the Percent
Find each percent. (For review, see pages 802 and 803.)
15. Five is what percent of 20?
16. What percent of 300 is 21?
17. What percent of 5 is 15?
18. Twelve is what percent of 60?
19. Sixteen is what percent of 10?
20. What percent of 50 is 37.5?

Foldables Study Organizer
Make this Foldable to help you organize information about solving linear equations. Begin with 4 sheets of plain \( \frac{8}{2} \) by 11" paper.

Step 1  Fold
Fold in half along the width.

Step 2  Open and Fold Again
Fold the bottom to form a pocket. Glue edges.

Step 3  Repeat Steps 1 and 2
Repeat three times and glue all four pieces together.

Step 4  Label
Label each pocket. Place an index card in each pocket.

Reading and Writing  As you read and study the chapter, you can write notes and examples on each index card.
WRITE EQUATIONS When writing equations, use variables to represent the unspecified numbers or measures referred to in the sentence or problem. Then write the verbal expressions as algebraic expressions. Some verbal expressions that suggest the equals sign are listed below.

- is
- equals
- is equal to
- is as much as
- is the same as
- is identical to

Example 1 Translate Sentences into Equations

Translate each sentence into an equation.

a. Five times the number \(a\) is equal to three times the sum of \(b\) and \(c\).

\[
\frac{5 \times a}{a} = \frac{3 \times (b + c)}{b + c}
\]

The equation is \(5a = 3(b + c)\).

b. Nine times \(y\) subtracted from 95 equals 37.

Rewrite the sentence so it is easier to translate.

\[
\frac{95 - 9y}{37} = \frac{37}{37}
\]

The equation is \(95 - 9y = 37\).
Using the **four-step problem-solving plan** can help you solve any word problem.

### Key Concept

**Four-Step Problem-Solving Plan**

1. **Step 1** Explore the problem.
2. **Step 2** Plan the solution.
3. **Step 3** Solve the problem.
4. **Step 4** Examine the solution.

Each step of the plan is important.

**Step 1 Explore the Problem**

To solve a verbal problem, first read the problem carefully and explore what the problem is about.
- Identify what information is given.
- Identify what you are asked to find.

**Step 2 Plan the Solution**

One strategy you can use to solve a problem is to write an equation. Choose a variable to represent one of the unspecified numbers in the problem. This is called **defining a variable**. Then use the variable to write expressions for the other unspecified numbers in the problem. *You will learn to use other strategies throughout this book.*

**Step 3 Solve the Problem**

Use the strategy you chose in Step 2 to solve the problem.

**Step 4 Examine the Solution**

Check your answer in the context of the original problem.
- Does your answer make sense?
- Does it fit the information in the problem?

### More About . . .

**Ice Cream**

The first ice cream plant was established in 1851 by Jacob Fussell. Today, 2,000,000 gallons of ice cream are produced in the United States each day.

*Source: World Book Encyclopedia*

### Example 2 Use the Four-Step Plan

**ICE CREAM** Use the information at the left. In how many days can 40,000,000 gallons of ice cream be produced in the United States?

**Explore**

You know that 2,000,000 gallons of ice cream are produced in the United States each day. You want to know how many days it will take to produce 40,000,000 gallons of ice cream.

**Plan**

Write an equation to represent the situation. Let \(d\) represent the number of days needed to produce the ice cream.

\[
\frac{2,000,000}{2,000,000} \times \frac{\text{the number of days}}{d} = \frac{40,000,000}{40,000,000}
\]

**Solve**

\[
2,000,000d = 40,000,000\]

Find \(d\) mentally by asking, "What number times 2,000,000 equals 40,000,000?"

\[
d = 20
\]

It will take 20 days to produce 40,000,000 gallons of ice cream.

**Examine**

If 2,000,000 gallons of ice cream are produced in one day, \(2,000,000 \times 20\) or 40,000,000 gallons are produced in 20 days. The answer makes sense.
A **formula** is an equation that states a rule for the relationship between certain quantities. Sometimes you can develop a formula by making a model.

**Algebra Activity**

**Surface Area**

- Mark each side of a rectangular box as the length \( \ell \), the width \( w \), or the height \( h \).
- Use scissors to cut the box so that each surface or face of the box is a separate piece.

**Analyze**

1. Write an expression for the area of the front of the box.
2. Write an expression for the area of the back of the box.
3. Write an expression for the area of one side of the box.
4. Write an expression for the area of the other side of the box.
5. Write an expression for the area of the top of the box.
6. Write an expression for the area of the bottom of the box.
7. The surface area of a rectangular box is the sum of all the areas of the faces of the box. If \( S \) represents surface area, write a formula for the surface area of a rectangular box.

**Make a Conjecture**

8. If \( s \) represents the length of the side of a cube, write a formula for the surface area of a cube.

---

**Example 3 Write a Formula**

Translate the sentence into a formula.

The perimeter of a rectangle equals two times the length plus two times the width.

**Words** Perimeter equals two times the length plus two times the width.

**Variables** Let \( P = \) perimeter, \( \ell = \) length, and \( w = \) width.

**Formula**

\[
P = 2\ell + 2w
\]

The formula for the perimeter of a rectangle is \( P = 2\ell + 2w \).

---

**Example 4 Translate Equations into Sentences**

Translate each equation into a verbal sentence.

a. \( 3m + 5 = 14 \)

Three times \( m \) plus five equals fourteen.
b. $w + v = y^2$

\[
\begin{array}{c}
w + v \\
= \\
y^2
\end{array}
\]

The sum of $w$ and $v$ equals the square of $y$.

**Example 5** Write a Problem

Write a problem based on the given information.

\[ a = \text{Rafael’s age} \quad a + 5 = \text{Tierra’s age} \quad a + 2(a + 5) = 46 \]

You know that $a$ represents Rafael’s age and $a + 5$ represents Tierra’s age. The equation adds $a$ plus twice ($a + 5$) to get 46. A sample problem is given below.

Tierra is 5 years older than Rafael. The sum of Rafael’s age and twice Tierra’s age equals 46. How old is Rafael?

---

**Check for Understanding**

**Concept Check**

1. List the four steps used in solving problems.
2. Analyze the following problem.
   
   *Misae has $1900 in the bank. She wishes to increase her account to a total of $3500 by depositing $30 per week from her paycheck. Will she reach her savings goal in one year?*
   
   a. How much money did Misae have in her account at the beginning?
   
   b. How much money will Misae add to her account in 10 weeks? in 20 weeks?
   
   c. Write an expression representing the amount added to the account after $w$ weeks have passed.
   
   d. What is the answer to the question? Explain.
3. **OPEN ENDED** Write a problem that can be answered by solving $x + 16 = 30$.

**Guided Practice**

Translate each sentence into an equation.

4. Two times a number $t$ decreased by eight equals seventy.
5. Five times the sum of $m$ and $n$ is the same as seven times $n$.

Translate each sentence into a formula.

6. The area $A$ of a triangle equals one half times the base $b$ times the height $h$.
7. The circumference $C$ of a circle equals the product of two, pi, and the radius $r$.

Translate each equation into a verbal sentence.

8. $14 + d = 6d$
9. $\frac{1}{3}b - \frac{3}{4} = 2a$

10. Write a problem based on the given information.

   \[c = \text{cost of a suit} \quad c - 25 = 150\]

**Application**

WRESTLING For Exercises 11 and 12, use the following information.

Darius is training to prepare for wrestling season. He weighs 155 pounds now. He wants to gain weight so that he starts the season weighing 160 pounds.

11. If $g$ represents the number of pounds he wants to gain, write an equation to represent the situation.
12. How many pounds does Darius need to gain to reach his goal?
Translate each sentence into an equation.

13. Two hundred minus three times \( x \) is equal to nine.
14. The sum of twice \( r \) and three times \( s \) is identical to thirteen.
15. The sum of one-third \( q \) and 25 is as much as twice \( q \).
16. The square of \( m \) minus the cube of \( n \) is sixteen.
17. Two times the sum of \( v \) and \( w \) is equal to two times \( z \).
18. Half of the sum of nine and \( p \) is the same as \( p \) minus three.
19. The number \( g \) divided by the number \( h \) is the same as seven more than twice the sum of \( g \) and \( h \).
20. Five-ninths the square of the sum of \( a \), \( b \), and \( c \) equals the sum of the square of \( a \) and the square of \( c \).

21. **GEOGRAPHY** The Pacific Ocean covers about 46% of Earth. If \( P \) represents the area of the Pacific Ocean and \( E \) represents the area of Earth, write an equation for this situation.

22. **GARDENING** Mrs. Patton is planning to place a fence around her vegetable garden. The fencing costs $1.75 per yard. She buys \( f \) yards of fencing and pays $3.50 in tax. If the total cost of the fencing is $73.50, write an equation to represent the situation.

Translate each sentence into a formula.

23. The area \( A \) of a parallelogram is the base \( b \) times the height \( h \).

24. The volume \( V \) of a pyramid is one-third times the product of the area of the base \( B \) and its height \( h \).

25. The perimeter \( P \) of a parallelogram is twice the sum of the lengths of the two adjacent sides, \( a \) and \( b \).

26. The volume \( V \) of a cylinder equals the product of \( \pi \), the square of the radius \( r \) of the base, and the height.

27. In a right triangle, the square of the measure of the hypotenuse \( c \) is equal to the sum of the squares of the measures of the legs, \( a \) and \( b \).

28. The temperature in degrees Fahrenheit \( F \) is the same as nine-fifths of the degrees Celsius \( C \) plus thirty-two.
Translate each equation into a verbal sentence.

29. \(d - 14 = 5\) 
30. \(2f + 6 = 19\) 
31. \(k^2 + 17 = 53 - j\)
32. \(2a = 7a - b\) 
33. \(\frac{3}{4}p + \frac{1}{2} = p\) 
34. \(\frac{2}{5}w = \frac{1}{2}w + 3\)
35. \(7(m + n) = 10n + 17\) 
36. \(4(t - s) = 5s + 12\)

37. **GEOMETRY** If \(a\) and \(b\) represent the lengths of the bases of a trapezoid and \(h\) represents its height, then the formula for the area \(A\) of the trapezoid is \(A = \frac{1}{2}h(a + b)\). Write the formula in words.

38. **SCIENCE** If \(r\) represents rate, \(t\) represents time, and \(d\) represents distance, then \(rt = d\). Write the formula in words.

Write a problem based on the given information.

39. \(y = \text{Yolanda’s height in inches}\) 
40. \(p = \text{price of a new backpack}\) 
\(y + 7 = \text{Lindsey’s height in inches}\) 
\(0.055p = \text{tax}\) 
\(2y + (y + 7) = 193\) 
\(p + 0.055p = 31.65\)

**GEOMETRY** For Exercises 41 and 42, use the following information.
The volume \(V\) of a cone equals one-third times the product of \(\pi\), the square of the radius \(r\) of the base, and the height \(h\).

41. Write the formula for the volume of a cone.
42. Find the volume of a cone if \(r\) is 10 centimeters and \(h\) is 30 centimeters.

**GEOMETRY** For Exercises 43 and 44, use the following information.
The volume \(V\) of a sphere is four-thirds times \(\pi\) times the radius \(r\) of the sphere cubed.

43. Write a formula for the volume of a sphere.
44. Find the volume of a sphere if \(r\) is 4 inches.

**LITERATURE** For Exercises 45–47, use the following information.
Edgar Rice Burroughs is the author of the *Tarzan of the Apes* stories. He published his first Tarzan story in 1912. Some years later, the town in southern California where he lived was named Tarzana.

45. Let \(y\) represent the number of years after 1912 that the town was named Tarzana. Write an expression for the year the town was named.
46. The town was named in 1928. Write an equation to represent the situation.
47. Use what you know about numbers to determine the number of years between the first Tarzan story and the naming of the town.

**TELEVISION** For Exercises 48–51, use the following information.
During a highly rated one-hour television program, the entertainment portion lasted 15 minutes longer than 4 times the advertising portion.

48. If \(a\) represents the time spent on advertising, write an expression for the entertainment portion.
49. Write an equation to represent the situation.
50. Use your equation and the guess-and-check strategy to determine the number of minutes spent on advertising. Choose different values of \(a\) and evaluate to find the solution.
51. Time the entertainment and advertising portions of a one-hour television program you like to watch. Describe what you found. Are the results of this problem similar to your findings?
52. **CRITICAL THINKING**  The surface area of a prism is the sum of the areas of the faces of the prism. Write a formula for the surface area of the triangular prism at the right.

53. **WRITING IN MATH**  Answer the question that was posed at the beginning of the lesson.

How are equations used to describe heights?
Include the following in your answer:
• an equation relating the Sears Tower, which is 1454 feet tall; the twin antenna towers on top of the building, which are \(a\) feet tall; and a total height, which is 1707 feet, and
• an equation representing the height of a building of your choice.

54. Which equation represents the following sentence?
*One fourth of a number plus five equals the number minus seven.*

A) \(\frac{1}{4}n + 7 = n - 5\)  
B) \(\frac{1}{4}n + 5 = n - 7\)  
C) \(4n + 7 = n - 5\)  
D) \(4n + 5 = n - 7\)

55. Which sentence can be represented by \(7(x + y) = 35?\)

A) Seven times \(x\) plus \(y\) equals 35.
B) One seventh of the sum of \(x\) and \(y\) equals 35.
C) Seven plus \(x\) and \(y\) equals 35.
D) Seven times the sum of \(x\) and \(y\) equals 35.

### Maintain Your Skills

#### Mixed Review

Find each square root. Use a calculator if necessary. Round to the nearest hundredth if the result is not a whole number or a simple fraction. \((Lesson 2-7)\)

56. \(\sqrt{8100}\)  
57. \(-\sqrt{\frac{25}{36}}\)  
58. \(\sqrt{90}\)  
59. \(-\sqrt{55}\)

Find the probability of each outcome if a die is rolled. \((Lesson 2-6)\)

60. a 6  
61. an even number  
62. a number greater than 2

Simplify each expression. \((Lesson 1-5)\)

63. \(12d + 3 - 4d\)  
64. \(7t^2 + t + 8t\)  
65. \(3(a + 2b) + 5a\)

Evaluate each expression. \((Lesson 1-2)\)

66. \(5(8 - 3) + 7 \cdot 2\)  
67. \(6(4^3 + 2^2)\)  
68. \(7(0.2 + 0.5) - 0.6\)

### Getting Ready for the Next Lesson

**PREREQUISITE SKILL**  Find each sum or difference. \((To review operations with fractions, see pages 798 and 799.\)

69. \(5.67 + 3.7\)  
70. \(0.57 + 2.8\)  
71. \(5.28 - 3.4\)  
72. \(9 - 7.35\)

73. \(\frac{2}{3} + \frac{1}{5}\)  
74. \(\frac{1}{6} + \frac{2}{3}\)  
75. \(\frac{7}{9} - \frac{2}{3}\)  
76. \(\frac{3}{4} - \frac{1}{6}\)
Solving Addition and Subtraction Equations

You can use algebra tiles to solve equations. To solve an equation means to find the value of the variable that makes the equation true. After you model the equation, the goal is to get the $x$ tile by itself on one side of the mat using the rules stated below.

### Rules for Equation Models

<table>
<thead>
<tr>
<th>Rule</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>You can remove or add the same number of identical algebra tiles to each side of the mat without changing the equation.</td>
<td><img src="image1" alt="Diagram" /></td>
</tr>
<tr>
<td>One positive tile and one negative tile of the same unit are a zero pair. Since $1 + (-1) = 0$, you can remove or add zero pairs to the equation mat without changing the equation.</td>
<td><img src="image2" alt="Diagram" /></td>
</tr>
</tbody>
</table>

### Use an equation model to solve $x - 3 = 2$.

**Step 1** Model the equation.

1. Place 1 $x$ tile and 3 negative 1 tiles on one side of the mat.
2. Place 2 positive 1 tiles on the other side of the mat.
3. Then add 3 positive 1 tiles to each side.

$$x - 3 = 2$$

$$x - 3 + 3 = 2 + 3$$

**Step 2** Isolate the $x$ term.

1. Group the tiles to form zero pairs.
2. Then remove all the zero pairs.
3. The resulting equation is $x = 5$.

### Model and Analyze

#### Use algebra tiles to solve each equation.

1. $x + 5 = 7$
2. $x + (-2) = 28$
3. $x + 4 = 27$
4. $x + (-3) = 4$
5. $x + 3 = -4$
6. $x + 7 = 2$

### Make a Conjecture

7. If $a = b$, what can you say about $a + c$ and $b + c$?
8. If $a = b$, what can you say about $a - c$ and $b - c$?
Solving Equations by Using Addition and Subtraction

What You’ll Learn

• Solve equations by using addition.
• Solve equations by using subtraction.

Vocabulary

• equivalent equation
• solve an equation

How can equations be used to compare data?

The graph shows some of the fastest-growing occupations from 1992 to 2005.

The difference between the percent of growth for medical assistants and the percent of growth for travel agents in these years is 5%. An equation can be used to find the percent of growth expected for medical assistants. If \( m \) is the percent of growth for medical assistants, then \( m - 66 = 5 \). You can use a property of equality to find the value of \( m \).

SOLVE USING ADDITION

Suppose your school’s boys’ soccer team has 15 members and the girls’ soccer team has 15 members. If each team adds 3 new players, the number of members on the boys’ and girls’ teams would still be equal.

\[
\begin{align*}
15 &= 15 & \text{Each team has 15 members before adding the new players.} \\
15 + 3 &= 15 + 3 & \text{Each team adds 3 new members.} \\
18 &= 18 & \text{Each team has 18 members after adding the new members.}
\end{align*}
\]

This example illustrates the **Addition Property of Equality**.

Key Concept

**Addition Property of Equality**

- **Words**: If an equation is true and the same number is added to each side, the resulting equation is true.
- **Symbols**: For any numbers \( a, b, \) and \( c \), if \( a = b \), then \( a + c = b + c \).
- **Examples**: 
  \[
  \begin{align*}
  7 &= 7 & 14 &= 14 \\
  7 + 3 &= 7 + 3 & 14 + (-6) &= 14 + (-6) \\
  10 &= 10 & 8 &= 8
  \end{align*}
  \]
If the same number is added to each side of an equation, then the result is an equivalent equation. **Equivalent equations** have the same solution.

\[ t + 3 = 5 \]  
**The solution of this equation is 2.**

\[ t + 3 + 2 = 5 + 2 \]  
**Using the Addition Property of Equality, add 2 to each side.**

\[ t + 5 = 7 \]  
**The solution of this equation is also 2.**

To **solve an equation** means to find all values of the variable that make the equation a true statement. One way to do this is to isolate the variable having a coefficient of 1 on one side of the equation. You can sometimes do this by using the Addition Property of Equality.

**Example 1** Solve by Adding a Positive Number

Solve \( m - 48 = 29 \). Then check your solution.

\[
\begin{align*}
    m - 48 & = 29 & \text{Original equation} \\
    m - 48 + 48 & = 29 + 48 & \text{Add 48 to each side.} \\
    m & = 77 & -48 + 48 = 0 \text{ and } 29 + 48 = 77 \\
\end{align*}
\]

To check that 77 is the solution, substitute 77 for \( m \) in the original equation.

**CHECK**

\[
\begin{align*}
    m & - 48 = 29 & \text{Original equation} \\
    77 & - 48 \not\leq 29 & \text{Substitute } 77 \text{ for } m. \\
    29 & = 29 \checkmark & \text{Subtract.} \\
\end{align*}
\]

The solution is 77.

**Example 2** Solve by Adding a Negative Number

Solve \( 21 + q = -18 \). Then check your solution.

\[
\begin{align*}
    21 + q & = -18 & \text{Original equation} \\
    21 + q + (-21) & = -18 + (-21) & \text{Add } -21 \text{ to each side.} \\
    q & = -39 & 21 + (-21) = 0 \text{ and } -18 + (-21) = -39 \\
\end{align*}
\]

**CHECK**

\[
\begin{align*}
    21 + q & = -18 & \text{Original equation} \\
    21 + (-39) \not\leq -18 & \text{Substitute } -39 \text{ for } q. \\
    -18 & = -18 \checkmark & \text{Add.} \\
\end{align*}
\]

The solution is -39.

**SOLVE USING SUBTRACTION** Similar to the Addition Property of Equality, there is a **Subtraction Property of Equality** that may be used to solve equations.
Example 3 Solve by Subtracting

Solve 142 + d = 97. Then check your solution.

142 + d = 97 \hspace{1cm} \text{Original equation}

142 + d − 142 = 97 − 142 \hspace{1cm} \text{Subtract 142 from each side.}

\[ d = -45 \]

142 − 142 = 0 and 97 − 142 = −45

CHECK 142 + d = 97 \hspace{1cm} \text{Original equation}

142 + (−45) \approx 97 \hspace{1cm} \text{Substitute −45 for } d.

\[ 97 = 97 \sqrt{\text{Add.}} \]

The solution is −45.

Remember that subtracting a number is the same as adding its inverse.

Example 4 Solve by Adding or Subtracting

Solve \( g + \frac{3}{4} = −\frac{1}{8} \) in two ways.

Method 1 Use the Subtraction Property of Equality.

\[ g + \frac{3}{4} = −\frac{1}{8} \hspace{1cm} \text{Original equation} \]

\[ g + \frac{3}{4} − \frac{3}{4} = −\frac{1}{8} − \frac{3}{4} \hspace{1cm} \text{Subtract } \frac{3}{4} \text{ from each side.} \]

\[ g = −\frac{7}{8} \]

\[ \frac{3}{4} − \frac{3}{4} = 0 \text{ and } −\frac{1}{8} − \frac{3}{4} = −\frac{1}{8} − \frac{6}{8} \text{ or } −\frac{7}{8} \]

The solution is \( −\frac{7}{8} \).

Method 2 Use the Addition Property of Equality.

\[ g + \frac{3}{4} = −\frac{1}{8} \hspace{1cm} \text{Original equation} \]

\[ g + \frac{3}{4} + \left(−\frac{3}{4}\right) = −\frac{1}{8} + \left(−\frac{3}{4}\right) \hspace{1cm} \text{Add } −\frac{3}{4} \text{ to each side.} \]

\[ g = −\frac{7}{8} \]

\[ \frac{3}{4} + \left(−\frac{3}{4}\right) = 0 \text{ and } −\frac{1}{8} + \left(−\frac{3}{4}\right) = −\frac{1}{8} + \left(−\frac{6}{8}\right) \text{ or } −\frac{7}{8} \]

The solution is \( −\frac{7}{8} \).

Example 5 Write and Solve an Equation

Write an equation for the problem. Then solve the equation and check your solution.

A number increased by 5 is equal to 42. Find the number.

\[ n + 5 = 42 \hspace{1cm} \text{Original equation} \]

\[ n + 5 − 5 = 42 − 5 \hspace{1cm} \text{Subtract 5 from each side.} \]

\[ n = 37 \]

\[ 5 − 5 = 0 \text{ and } 42 − 5 = 37 \]

CHECK \[ n + 5 = 42 \hspace{1cm} \text{Original equation} \]

\[ 37 + 5 \nleq 42 \hspace{1cm} \text{Substitute 37 for } n. \]

\[ 42 = 42 \sqrt{\text{Add.}} \]

The solution is 37.
Lesson 3-2  Solving Equations by Using Addition and Subtraction

**Example 6 Write an Equation to Solve a Problem**

**HISTORY** Refer to the information at the right.

In the fourteenth century, the part of the Great Wall of China that was built during Qui Shi Huangdi’s time was repaired, and the wall was extended. When the wall was completed, it was 2500 miles long. How much of the wall was added during the 1300s?

**Words**  The original length plus the additional length equals 2500.

**Variable** Let \( a \) = the additional length.

**Equation**

\[
1000 + a = 2500
\]

1. **Original equation**
2. Subtract 1000 from each side.
3. \( a = 1500 \)

The Great Wall of China was extended 1500 miles in the 1300s.

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**Check for Understanding**

**Concept Check**

1. **OPEN ENDED** Write three equations that are equivalent to \( n + 14 = 27 \).
2. **Compare and contrast** the Addition Property of Equality and the Subtraction Property of Equality.
3. **Show** two ways to solve \( g + 94 = 75 \).

**Guided Practice**

Solve each equation. Then check your solution.

4. \( t - 4 = -7 \)  
5. \( p + 19 = 6 \)  
6. \( 15 + r = 71 \)
7. \( 104 = y - 67 \)  
8. \( h - 0.78 = 2.65 \)  
9. \( \frac{2}{3} + w = 1\frac{1}{2} \)

Write an equation for each problem. Then solve the equation and check your solution.

10. Twenty-one subtracted from a number is \(-8\). Find the number.
11. A number increased by \(-37\) is \(-91\). Find the number.

**Application**

**CARS** For Exercises 12–14, use the following information.

The average time it takes to manufacture a car in the United States is equal to the average time it takes to manufacture a car in Japan plus 8.1 hours. The average time it takes to manufacture a car in the United States is 24.9 hours.

12. Write an addition equation to represent the situation.
13. What is the average time to manufacture a car in Japan?
14. The average time it takes to manufacture a car in Europe is 35.5 hours. What is the difference between the average time it takes to manufacture a car in Europe and the average time it takes to manufacture a car in Japan?
Solve each equation. Then check your solution.

15. \( v - 9 = 14 \)
16. \( s - 19 = -34 \)
17. \( g + 5 = 33 \)
18. \( 18 + z = 44 \)
19. \( a - 55 = -17 \)
20. \( t - 72 = -44 \)
21. \( -18 = -61 + d \)
22. \( -25 = -150 + q \)
23. \( r - (-19) = -77 \)
24. \( b - (-65) = 15 \)
25. \( 18 - (-f) = 91 \)
26. \( 125 - (-u) = 88 \)
27. \( -2.56 + c = 0.89 \)
28. \( k + 0.6 = -3.84 \)
29. \( -6 = m + (-3.42) \)
30. \( 6.2 = -4.83 + y \)
31. \( t - 8.5 = 7.15 \)
32. \( q - 2.78 = 4.2 \)
33. \( x - \frac{3}{4} = \frac{5}{6} \)
34. \( a - \frac{3}{5} = -\frac{7}{10} \)
35. \( -\frac{1}{2} + p = \frac{5}{8} \)
36. \( \frac{2}{3} + r = -\frac{4}{9} \)
37. \( \frac{2}{3} = v + \frac{4}{5} \)
38. \( \frac{2}{5} = w + \frac{3}{4} \)

39. If \( x - 7 = 14 \), what is the value of \( x - 2 \)?
40. If \( t + 8 = -12 \), what is the value of \( t + 1 \)?

**GEOMETRY**  For Exercises 41 and 42, use the rectangle at the right.

41. Write an equation you could use to solve for \( x \) and then solve for \( x \).
42. Write an equation you could use to solve for \( y \) and then solve for \( y \).

Write an equation for each problem. Then solve the equation and check your solution.

43. Eighteen subtracted from a number equals 31. Find the number.
44. What number decreased by 77 equals \( -18 \)?
45. A number increased by \( -16 \) is \( -21 \). Find the number.
46. The sum of a number and \( -43 \) is 102. What is the number?
47. What number minus one-half is equal to negative three-fourths?
48. The sum of 19 and 42 and a number is equal to 87. What is the number?
49. Determine whether \( x + x = x \) is sometimes, always, or never true. Explain your reasoning.
50. Determine whether \( x + 0 = x \) is sometimes, always, or never true. Explain your reasoning.

**GAS MILEAGE**  For Exercises 51–55, use the following information.
A midsize car with a 4-cylinder engine goes 10 miles more on a gallon of gasoline than a luxury car with an 8-cylinder engine. A midsize car consumes one gallon of gas for every 34 miles driven.

51. Write an addition equation to represent the situation.
52. How many miles does a luxury car travel on a gallon of gasoline?
53. A subcompact car with a 3-cylinder engine goes 13 miles more than a luxury car on one gallon of gas. How far does a subcompact car travel on a gallon of gasoline?
54. How many more miles does a subcompact travel on a gallon of gasoline than a midsize car?
55. Estimate how many miles a full-size car with a 6-cylinder engine goes on one gallon of gasoline. Explain your reasoning.
**HISTORY**  For Exercises 56 and 57, use the following information.
Over the years, the height of the Great Pyramid at Giza, Egypt, has decreased.

56. Write an addition equation to represent the situation.

57. What was the decrease in the height of the pyramid?

**LIBRARIES**  For Exercises 58–61, use the graph at the right to write an equation for each situation. Then solve the equation.

58. How many more volumes does the Library of Congress have than the Harvard University Library?

59. How many more volumes does the Harvard University Library have than the New York Public Library?

60. How many more volumes does the Library of Congress have than the New York Public Library?

61. What is the total number of volumes in the three largest U.S. libraries?

**ANIMALS**  For Exercises 62–64, use the information below to write an equation for each situation. Then solve the equation.
Wildlife authorities monitor the population of animals in various regions. One year’s deer population in Dauphin County, Pennsylvania, is shown in the graph below.

62. How many more newborns are there than one-year-olds?

63. How many more females are there than males?

64. What is the total deer population?
65. **CRITICAL THINKING** If \( a - b = x \), what values of \( a \), \( b \), and \( x \) would make the equation \( a + x = b + x \) true?

66. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can equations be used to compare data?

Include the following in your answer:

- an explanation of how to solve the equation to find the growth rate for medical assistants, and
- a sample problem and related equation using the information in the graph.

67. Which equation is not equivalent to \( b - 15 = 32 \)?

- **A** \( b + 5 = 52 \)
- **B** \( b - 20 = 27 \)
- **C** \( b - 13 = 30 \)
- **D** \( b = 47 \)

68. What is the solution of \( x - 167 = -52 \)?

- **A** 115
- **B** -115
- **C** 219
- **D** -219

### Maintain Your Skills

**Mixed Review**

**GEOMETRY** For Exercises 69 and 70, use the following information.

The area of a circle is the product of \( \pi \) times the radius \( r \) squared. *(Lesson 3-1)*

69. Write the formula for the area of the circle.

70. If a circle has a radius of 16 inches, find its area.

Replace each \( \circ \) with \( > \), \( < \), or \( = \) to make the sentence true. *(Lesson 2-7)*

- 71. \( \frac{1}{2} \circ \sqrt{2} \)
- 72. \( \frac{3}{4} \circ \frac{2}{3} \)
- 73. 0.375 \( \circ \frac{3}{8} \)

Use each set of data to make a stem-and-leaf plot. *(Lesson 2-5)*

- 74. 54, 52, 43, 41, 40, 36, 35, 31, 32, 34, 42, 56
- 75. 2.3, 1.4, 1.7, 1.2, 2.6, 0.8, 0.5, 2.8, 4.1, 2.9, 4.5, 1.1

Identify the hypothesis and conclusion of each statement. *(Lesson 1-7)*

- 76. For \( y = 2 \), \( 4y - 6 = 2 \).
- 77. There is a science quiz every Friday.

Evaluate each expression. Name the property used in each step. *(Lesson 1-4)*

- 78. \( 4(16 ÷ 4^2) \)
- 79. \( (2^5 - 5^2) + (4^2 - 2^4) \)

Find the solution set for each inequality, given the replacement set. *(Lesson 1-3)*

- 80. \( 3x + 2 > 2; \{0, 1, 2\} \)
- 81. \( 2y^2 - 1 > 0; \{1, 3, 5\} \)

**Getting Ready for the Next Lesson**

**PREREQUISITE SKILL** Find each product or quotient. *(To review operations with fractions, see pages 800 and 801.)*

- 82. \( 6.5 \times 2.8 \)
- 83. \( 70.3 \times 0.15 \)
- 84. \( 17.8 ÷ 2.5 \)
- 85. \( 0.33 + 1.5 \)
- 86. \( \frac{2}{3} \times \frac{5}{8} \)
- 87. \( \frac{5}{9} \times \frac{3}{10} \)
- 88. \( \frac{1}{2} + \frac{2}{5} \)
- 89. \( \frac{8}{9} + \frac{4}{15} \)
Lesson 3-3
Solving Equations by Using Multiplication and Division

What You’ll Learn

- Solve equations by using multiplication.
- Solve equations by using division.

How can equations be used to find how long it takes light to reach Earth?

It may look like all seven stars in the Big Dipper are the same distance from Earth, but in fact, they are not. The diagram shows the distance between each star and Earth.

Light travels at a rate of about 5,870,000,000,000 miles per year. In general, the rate at which something travels times the time equals the distance \((rt = d)\). The following equation can be used to find the time it takes light to reach Earth from the closest star in the Big Dipper.

\[
rt = d
\]

\[
5,870,000,000,000t = 311,110,000,000,000
\]

SOLVE USING MULTIPLICATION

To solve equations such as the one above, you can use the Multiplication Property of Equality.

Key Concept

**Multiplication Property of Equality**

- **Words**
  If an equation is true and each side is multiplied by the same number, the resulting equation is true.
- **Symbols**
  For any numbers \(a\), \(b\), and \(c\), if \(a = b\), then \(ac = bc\).
- **Examples**
  
  \[
  \begin{align*}
  6 &= 6 \\
  9 &= 9 \\
  10 &= 10 \\
  6 \times 2 &= 6 \times 2 \\
  9 \times (-3) &= 9 \times (-3) \\
  10 \times \frac{1}{2} &= 10 \times \frac{1}{2} \\
  12 &= 12 \\
  -27 &= -27 \\
  5 &= 5
  \end{align*}
  \]

Example 1

Solve Using Multiplication by a Positive Number

Solve \(\frac{t}{30} = \frac{7}{10}\). Then check your solution.

\[
\frac{t}{30} = \frac{7}{10} \quad \text{Original equation}
\]

\[
30\left(\frac{t}{30}\right) = 30\left(\frac{7}{10}\right) \quad \text{Multiply each side by 30.}
\]

\[
t = 21
\]

(continued on the next page)
Space Travel

On July 20, 1969, Neil Armstrong stepped on the surface of the moon. On the moon, his suit and life-support backpacks weighed about 33 pounds.}

Source: NASA

### Example 2 Solve Using Multiplication by a Fraction

Solve \( \left(2\frac{1}{4}\right)g = 1\frac{1}{2} \).

1. Original equation:
   \[
   \left(2\frac{1}{4}\right)g = 1\frac{1}{2}
   \]
2. Rewrite each mixed number as an improper fraction:
   \[
   \left(\frac{9}{4}\right)g = \frac{3}{2}
   \]
3. Multiply each side by \(\frac{4}{9} \), the reciprocal of \(\frac{9}{4} \):
   \[
   g = \frac{12}{18} \text{ or } \frac{2}{3}
   \]
   Check this result.

The solution is \(\frac{2}{3}\).

### Example 3 Solve Using Multiplication by a Negative Number

Solve \( 42 = -6m \).

1. Original equation:
   \[
   42 = -6m
   \]
2. Multiply each side by \(-\frac{1}{6} \), the reciprocal of \(-6 \):
   \[
   -7 = m
   \]
   Check this result.

The solution is \(-7\).

You can write an equation to represent a real-world problem. Then use the equation to solve the problem.

### Example 4 Write and Solve an Equation Using Multiplication

**SPACE TRAVEL** Refer to the information about space travel at the left. The weight of anything on the moon is about one-sixth its weight on Earth. What was the weight of Neil Armstrong’s suit and life-support backpacks on Earth?

**Words** One sixth times the weight on Earth equals the weight on the moon.

**Variable** Let \(w\) = the weight on Earth.

<table>
<thead>
<tr>
<th>One sixth</th>
<th>times</th>
<th>the weight on Earth</th>
<th>equals</th>
<th>the weight on the moon</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{1}{6})</td>
<td>(w)</td>
<td>=</td>
<td>33</td>
<td></td>
</tr>
</tbody>
</table>

1. Original equation:
   \[
   \frac{1}{6}w = 33
   \]
2. Multiply each side by 6:
   \[
   6\left(\frac{1}{6}w\right) = 6(33)
   \]
   \[
   w = 198
   \]

The weight of Neil Armstrong’s suit and life-support backpacks on Earth was about 198 pounds.
**SOLVE USING DIVISION** The equation in Example 3, $42 = -6m$, was solved by multiplying each side by $-\frac{1}{6}$. The same result could have been obtained by dividing each side by $-6$. This method uses the **Division Property of Equality**.

### Key Concept — Division Property of Equality

- **Words** If an equation is true and each side is divided by the same nonzero number, the resulting equation is true.
- **Symbols** For any numbers $a$, $b$, and $c$, with $c \neq 0$, if $a/b = b/c$, then $a = b$.
- **Examples**

<table>
<thead>
<tr>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>15</td>
<td>28</td>
</tr>
<tr>
<td>15</td>
<td>15</td>
<td>28</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>-7</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>-4</td>
</tr>
</tbody>
</table>

### Example 5 — Solve Using Division by a Positive Number

Solve $13s = 195$. Then check your solution.

1. $13s = 195$  
   Original equation
2. \[ \frac{13s}{13} = \frac{195}{13} \] Divide each side by 13.
3. \[ s = 15 \] \[ \frac{13s}{13} = s \text{ and } \frac{195}{13} = 15 \]

**CHECK**

1. $13s = 195$  
   Original equation
2. \[ 13(15) \neq 195 \] Substitute 15 for $s$.
3. \[ 195 = 195 \checkmark \]

The solution is 15.

### Example 6 — Solve Using Division by a Negative Number

Solve $-3x = 12$.

1. $-3x = 12$  
   Original equation
2. \[ \frac{-3x}{-3} = \frac{12}{-3} \] Divide each side by $-3$.
3. \[ x = -4 \] \[ -\frac{3x}{3} = x \text{ and } -\frac{12}{-3} = -4 \]

The solution is $-4$.

### Example 7 — Write and Solve an Equation Using Division

Write an equation for the problem below. Then solve the equation.

Negative eighteen times a number equals $-198$.

\[ -18n = -198 \]  
Original equation
\[ \frac{-18n}{-18} = \frac{-198}{-18} \] Divide each side by $-18$.
\[ n = 11 \]  
Check this result.

The solution is 11.
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Concept Check

1. OPEN ENDED  Write a multiplication equation that has a solution of \(-3\).

2. Explain why the Multiplication Property of Equality and the Division Property of Equality can be considered the same property.

3. FIND THE ERROR  Casey and Juanita are solving \(8n = -72\).

\[
\begin{align*}
\text{Casey} & : \quad 8n = -72 \\
& \implies 8n(8) = -72(8) \\
& \implies n = -576 \\
\text{Juanita} & : \quad 8n = -72 \\
& \implies \frac{8n}{8} = \frac{-72}{8} \\
& \implies n = -9
\end{align*}
\]

Who is correct? Explain your reasoning.

Guided Practice

Solve each equation. Then check your solution.

4. \(-2g = -84\)
5. \(\frac{t}{7} = -5\)
6. \(\frac{a}{36} = \frac{4}{9}\)
7. \(\frac{4}{5}k = \frac{8}{9}\)
8. \(3.15 = 1.5y\)
9. \(\left(3\frac{1}{4}\right)p = 2\frac{1}{2}\)

Write an equation for each problem. Then solve the equation.

10. Five times a number is 120. What is the number?
11. Two fifths of a number equals \(-24\). Find the number.

Application

12. GEOGRAPHY  The discharge of a river is defined as the width of the river times the average depth of the river times the speed of the river. At one location in St. Louis, the Mississippi River is 533 meters wide, its speed is 0.6 meter per second, and its discharge is 3198 cubic meters per second. How deep is the Mississippi River at this location?

\[
\text{Discharge} = \text{Width} \times \text{Depth} \times \text{Speed}
\]

\[
0.6 \text{ m/s} \times 533 \text{ m} \times \text{Depth} = 3198 \text{ m}^3/\text{s}
\]

Practice and Apply

Solve each equation. Then check your solution.

13. \(-5r = 55\)
14. \(8d = 48\)
15. \(-910 = -26a\)
16. \(-1634 = 86s\)
17. \(\frac{b}{7} = -11\)
18. \(-\frac{v}{5} = -45\)
19. \(\frac{2}{3}n = 14\)
20. \(\frac{2}{5}g = -14\)
21. \(\frac{5}{24} = \frac{5}{24}\)
22. \(\frac{2}{45} = \frac{2}{5}\)
23. \(1.9f = -11.78\)
24. \(0.49k = 6.272\)
25. \(-2.8m = 9.8\)
26. \(-5.73q = 97.41\)
27. \(\left(-2\frac{3}{5}\right)t = -22\)
28. \(\left(3\frac{2}{3}\right)x = -5\frac{1}{2}\)
29. \(-5h = -3\frac{2}{3}\)
30. \(3p = 4\frac{1}{5}\)

31. If \(4m = 10\), what is the value of \(12m\)?
32. If \(15b = 55\), what is the value of \(3b\)?

Homework Help

For See Exercises Examples
13–32 1–3, 5, 6
33–38 7
39–49 4

Extra Practice
See page 826.
Write an equation for each problem. Then solve the equation.

33. Seven times a number equals $-84$. What is the number?
34. Negative nine times a number is $-117$. Find the number.
35. One fifth of a number is $12$. Find the number.
36. Negative three eighths times a number equals $12$. What is the number?
37. Two and one half times a number equals one and one fifth. Find the number.
38. One and one third times a number is $-4.82$. What is the number?

**GENETICS**  For Exercises 39–41, use the following information.
Research conducted by a daily U.S. newspaper has shown that about one seventh of people in the world are left-handed.

39. Write a multiplication equation relating the number of left-handed people $\ell$ and the total number of people $p$.
40. About how many left-handed people are there in a group of $350$ people?
41. If there are $65$ left-handed people in a group, about how many people are in that group?

**WORLD RECORDS**  In 1993, a group of people in Utica, New York, made a very large round jelly doughnut which broke the world record for doughnut size. It weighed $1.5$ tons and had a circumference of $50$ feet. What was the diameter of the doughnut? (Hint: $C = \pi d$)

**BASEBALL**  For Exercises 43–45, use the following information.
In baseball, if all other factors are the same, the speed of a four-seam fastball is faster than a two-seam fastball. The distance from the pitcher’s mound to home plate is $60.5$ feet.

43. How long does it take a two-seam fastball to go from the pitcher’s mound to home plate? Round to the nearest hundredth. (Hint: $rt = d$)
44. How long does it take a four-seam fastball to go from the pitcher’s mound to home plate? Round to the nearest hundredth.
45. How much longer does it take for a two-seam fastball to reach home plate than a four-seam fastball?

**PHYSICAL SCIENCE**  For Exercises 46–49, use the following information.
In science lab, Devin and his classmates are asked to determine how many grams of hydrogen and how many grams of oxygen are in $477$ grams of water. Devin used what he learned in class to determine that for every $8$ grams of oxygen in water, there is $1$ gram of hydrogen.

46. If $x$ represents the number of grams of hydrogen, write an expression to represent the number of grams of oxygen.
47. Write an equation to represent the situation.
48. How many grams of hydrogen are in $477$ grams of water?
49. How many grams of oxygen are in $477$ grams of water?

50. **CRITICAL THINKING**  If $6y - 7 = 4$, what is the value of $18y - 21$?
51. **Writing in Math** Answer the question that was posed at the beginning of the lesson.

How can equations be used to find how long it takes light to reach Earth?
Include the following in your answer:
- an explanation of how to find the length of time it takes light to reach Earth from the closest star in the Big Dipper, and
- an equation describing the situation for the farthest star in the Big Dipper.

52. The rectangle at the right is divided into 5 identical squares. If the perimeter of the rectangle is 48 inches, what is the area of each square?

- A 4 in\(^2\)
- B 9.8 in\(^2\)
- C 16 in\(^2\)
- D 23.04 in\(^2\)

53. Which equation is equivalent to \(4t/110 = 20\)?

- A \(-2t = -10\)
- B \(t = 80\)
- C \(2t = 5\)
- D \(-8t = 40\)

---

**Maintain Your Skills**

### Mixed Review

Solve each equation. Then check your solution.  
(Lesson 3-2)

54. \(m + 14 = 81\)  
55. \(d - 27 = -14\)  
56. \(17 - (-w) = -55\)

57. Translate the following sentence into an equation.  
(Lesson 3-1)

Ten times a number \(a\) is equal to 5 times the sum of \(b\) and \(c\).

Find each product.  
(Lesson 2-3)

58. \((-5)(12)\)  
59. \((-2.93)(-0.003)\)  
60. \((-4)(0)(-2)(-3)\)

Graph each set of numbers on a number line.  
(Lesson 2-1)

61. \{-4, -3, -1, 3\}  
62. \{integers between -6 and 10\}  
63. \{integers less than -4\}  
64. \{integers less than 0 and greater than -6\}

Name the property illustrated by each statement.  
(Lesson 1-6)

65. \(67 + 3 = 3 + 67\)  
66. \((5 \cdot m) \cdot n = 5 \cdot (m \cdot n)\)

---

**Getting Ready for the Next Lesson**

**PREREQUISITE SKILL** Use the order of operations to find each value.  
(To review the order of operations, see Lesson 1-2.)

67. \(2 \times 8 + 9\)  
68. \(24 \div 3 - 8\)  
69. \(\frac{3}{8}(17 + 7)\)  
70. \(\frac{15 - 9}{26 + 12}\)

---

**Practice Quiz 1**

**GEOMETRY** For Exercises 1 and 2, use the following information. The surface area \(S\) of a sphere equals four times \(\pi\) times the square of the radius \(r\).  
(Lesson 3-1)

1. Write the formula for the surface area of a sphere.
2. What is the surface area of a sphere if the radius is 7 centimeters?

Solve each equation. Then check your solution.  
(Lessons 3-2 and 3-3)

3. \(d + 18 = -27\)  
4. \(m - 77 = -61\)  
5. \(-12 + a = -36\)  
6. \(t - (-16) = 9\)  
7. \(\frac{2}{3}p = 18\)  
8. \(-17y = 391\)  
9. \(5x = -45\)  
10. \(-\frac{2}{5}d = -10\)
Solving Multi-Step Equations

You can use an equation model to solve multi-step equations.

Solve $3x + 5 = -7$.

**Step 1** Model the equation.

\[3x + 5 = -7\]

Place $3 \times$ tiles and 5 positive 1 tiles on one side of the mat. Place 7 negative 1 tiles on the other side of the mat.

**Step 2** Isolate the $x$ term.

\[3x + 5 - 5 = -7 - 5\]

Since there are 5 positive 1 tiles with the $x$ tiles, add 5 negative 1 tiles to each side to form zero pairs.

**Step 3** Remove zero pairs.

\[3x = -12\]

Group the tiles to form zero pairs and remove the zero pairs.

**Step 4** Group the tiles.

\[\frac{3x}{3} = \frac{-12}{3}\]

Separate the tiles into 3 equal groups to match the $3 \times$ tiles. Each $x$ tile is paired with 4 negative 1 tiles. Thus, $x = -4$.

**Model** Use algebra tiles to solve each equation.

1. $2x - 3 = -9$
2. $3x + 5 = 14$
3. $3x - 2 = 10$
4. $-8 = 2x + 4$
5. $3 + 4x = 11$
6. $2x + 7 = 1$
7. $9 = 4x - 7$
8. $7 + 3x = -8$
9. **MAKE A CONJECTURE** What steps would you use to solve $7x - 12 = -61$?
### 3-4 Solving Multi-Step Equations

**What You’ll Learn**

- Solve problems by working backward.
- Solve equations involving more than one operation.

**Vocabulary**

- work backward
- multi-step equations
- consecutive integers
- number theory

**How can equations be used to estimate the age of an animal?**

An American alligator hatchling is about 8 inches long. These alligators grow about 12 inches per year. Therefore, the expression $8 + 12a$ represents the length in inches of an alligator that is $a$ years old.

Since 10 feet 4 inches equals 10(12) + 4 or 124 inches, the equation $8 + 12a = 124$ can be used to estimate the age of the alligator in the photograph. Notice that this equation involves more than one operation.

** WORK BACKWARD ** Work backward is one of many problem-solving strategies that you can use. Here are some other problem-solving strategies.

<table>
<thead>
<tr>
<th>Problem-Solving Strategies</th>
<th>Work backward is one of many problem-solving strategies that you can use. Here are some other problem-solving strategies.</th>
</tr>
</thead>
<tbody>
<tr>
<td>draw a diagram</td>
<td>create a simpler (or similar) diagram to help visualize the problem.</td>
</tr>
<tr>
<td>make a table or chart</td>
<td>create a chart to organize and track the steps of the problem.</td>
</tr>
<tr>
<td>make a model</td>
<td>create a model or diagram to help visualize the problem.</td>
</tr>
<tr>
<td>guess and check</td>
<td>make a guess and check the answer to see if it fits the problem.</td>
</tr>
<tr>
<td>check for hidden assumptions</td>
<td>look for hidden assumptions in the problem.</td>
</tr>
<tr>
<td>use a graph</td>
<td>identify the subgoals.</td>
</tr>
</tbody>
</table>

**Example 1 Work Backward to Solve a Problem**

Solve the following problem by working backward.

After cashing her paycheck, Tara paid her father the $20 she had borrowed. She then spent half of the remaining money on a concert ticket. She bought lunch for $4.35 and had $10.55 left. What was the amount of the paycheck?

Start at the end of the problem and undo each step.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Undo the Statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>She had $10.55 left.</td>
<td>$10.55</td>
</tr>
<tr>
<td>She bought lunch for $4.35.</td>
<td>$10.55 + $4.35 = $14.90</td>
</tr>
<tr>
<td>She spent half of the money on a concert ticket.</td>
<td>$14.90 × 2 = $29.80</td>
</tr>
<tr>
<td>She paid her father $20.</td>
<td>$29.80 + $20.00 = $49.80</td>
</tr>
</tbody>
</table>

The paycheck was for $49.80. Check this answer in the context of the problem.
SOLVE MULTI-STEP EQUATIONS  To solve equations with more than one operation, often called **multi-step equations**, undo operations by working backward.

**Example 2  Solve Using Addition and Division**

Solve $7m - 17 = 60$. Then check your solution.

- $7m - 17 = 60$  **Original equation**
- $7m - 17 + 17 = 60 + 17$  Add 17 to each side.
- $7m = 77$  **Simplify**.
- $\frac{7m}{7} = \frac{77}{7}$  **Divide each side by 7**.
- $m = 11$  **Simplify**.

**CHECK**

- $7m - 17 = 60$  **Original equation**
- $7(11) - 17 \not\leq 60$  Substitute 11 for $m$.
- $77 - 17 \not\leq 60$  **Multiply**.
- $60 = 60 \checkmark$  The solution is 11.

You have seen a multi-step equation in which the first, or **leading**, coefficient is an integer. You can use the same steps if the leading coefficient is a fraction.

**Example 3  Solve Using Subtraction and Multiplication**

Solve $\frac{t}{8} + 21 = 14$. Then check your solution.

- $\frac{t}{8} + 21 = 14$  **Original equation**
- $\frac{t}{8} + 21 - 21 = 14 - 21$  Subtract 21 from each side.
- $\frac{t}{8} = -7$  **Simplify**.
- $8\left(\frac{t}{8}\right) = 8(-7)$  **Multiply each side by 8**.
- $t = -56$  **Simplify**.

**CHECK**

- $\frac{t}{8} + 21 = 14$  **Original equation**
- $-\frac{56}{8} + 21 \not\leq 14$  Substitute $-56$ for $t$.
- $-7 + 21 \not\leq 14$  **Divide**.
- $14 = 14 \checkmark$  The solution is $-56$.

**Example 4  Solve Using Multiplication and Addition**

Solve $\frac{p - 15}{9} = -6$.

- $\frac{p - 15}{9} = -6$  **Original equation**
- $9\left(\frac{p - 15}{9}\right) = 9(-6)$  **Multiply each side by 9**.
- $p - 15 = -54$  **Simplify**.
- $p - 15 + 15 = -54 + 15$  **Add 15 to each side**.
- $p = -39$  The solution is $-39$.
**Example 5 Write and Solve a Multi-Step Equation**

Write an equation for the problem below. Then solve the equation.

*Two-thirds of a number minus six is −10.*

\[
\frac{2}{3}n - 6 = -10 \quad \text{Original equation}
\]

\[
\frac{2}{3}n - 6 + 6 = -10 + 6 \quad \text{Add 6 to each side.}
\]

\[
\frac{2}{3}n = -4 \quad \text{Simplify.}
\]

\[
\frac{3}{2}\left(\frac{2}{3}n\right) = \frac{3}{2}(-4) \quad \text{Multiply each side by} \quad \frac{3}{2}.
\]

\[
n = -6 \quad \text{Simplify.}
\]

The solution is \(-6\).

**Consecutive integers** are integers in counting order, such as 7, 8, and 9. Beginning with an even integer and counting by two will result in *consecutive even integers*. For example, \(-4, -2, 0,\) and \(2\) are consecutive even integers. Beginning with an odd integer and counting by two will result in *consecutive odd integers*. For example, \(-3, -1, 1, 3\) and \(5\) are consecutive odd integers. The study of numbers and the relationships between them is called **number theory**.

**Example 6 Solve a Consecutive Integer Problem**

**NUMBER THEORY** Write an equation for the problem below. Then solve the equation and answer the problem.

*Find three consecutive even integers whose sum is \(-42\).*

Let \(n\) = the least even integer.

Then \(n + 2\) = the next greater even integer, and

\(n + 4\) = the greatest of the three even integers.

\[
\text{The sum of three consecutive even integers} \quad \frac{n + (n + 2) + (n + 4)}{} = \frac{-42}{-42}
\]

\[
n + (n + 2) + (n + 4) = -42 \quad \text{Original equation}
\]

\[
3n + 6 = -42 \quad \text{Simplify.}
\]

\[
3n + 6 - 6 = -42 - 6 \quad \text{Subtract 6 from each side.}
\]

\[
3n = -48 \quad \text{Simplify.}
\]

\[
\frac{3n}{3} = \frac{-48}{3} \quad \text{Divide each side by 3}
\]

\[
n = -16 \quad \text{Simplify.}
\]

\[
n + 2 = -16 + 2 \quad \text{or} \quad -14 \quad \text{or} \quad n + 4 = -16 + 4 \quad \text{or} \quad -12
\]

The consecutive even integers are \(-16, -14,\) and \(-12\).

**CHECK** \(-16, -14,\) and \(-12\) are consecutive even integers.

\[
-16 + (-14) + (-12) = -42 \quad \text{✓}
\]
Check for Understanding

**Concept Check**

1. **OPEN ENDED** Give two examples of multi-step equations that have a solution of \(-2\).

2. List the steps used to solve \(\frac{w + 3}{5} - 4 = 6\).

3. Write an expression for the odd integer before odd integer \(n\).

4. Justify each step.
   \[
   \frac{4 - 2d}{5} + 3 = 9
   \]
   \[
   \frac{4 - 2d}{5} + 3 - 3 = 9 - 3
   \]
   \[
   \frac{4 - 2d}{5} = 6
   \]
   \[
   \frac{4 - 2d}{5} (5) = 6(5)
   \]
   \[
   4 - 2d = 30
   \]
   \[
   4 - 2d - 4 = 30 - 4
   \]
   \[
   -2d = 26
   \]
   \[
   -2d = 26
   \]
   \[
   d = -13
   \]

5. Solve each problem by working backward.
   5. A number is multiplied by seven, and then the product is added to 13. The result is 55. What is the number?
   6. **LIFE SCIENCE** A bacteria population triples in number each day. If there are 2,187,000 bacteria on the seventh day, how many bacteria were there on the first day?

6. Solve each equation. Then check your solution.
   7. \(4g - 2 = -6\)
   8. \(18 = 5p + 3\)
   9. \(\frac{3}{2}n - 8 = 11\)
   10. \(\frac{b + 4}{-2} = -17\)
   11. \(0.2n + 3 = 8.6\)
   12. \(3.1y - 1.5 = 5.32\)

7. Write an equation and solve each problem.
   13. Twelve decreased by twice a number equals \(-34\). Find the number.
   14. Find three consecutive integers whose sum is 42.

8. **APPLICATION**

15. **WORLD CULTURES** The English alphabet contains 2 more than twice as many letters as the Hawaiian alphabet. How many letters are there in the Hawaiian alphabet?

**Practice and Apply**

Solve each problem by working backward.

16. A number is divided by 4, and then the quotient is added to 17. The result is 25. Find the number.

17. Nine is subtracted from a number, and then the difference is multiplied by 5. The result is 75. What is the number?
Solve each problem by working backward.

18. **GAMES** In the Trivia Bowl, each finalist must answer four questions correctly. Each question is worth twice as much as the question before it. The fourth question is worth $6000. How much is the first question worth?

19. **ICE SCULPTING** Due to melting, an ice sculpture loses one-half its weight every hour. After 8 hours, it weighs \(\frac{5}{16}\) of a pound. How much did it weigh in the beginning?

20. **FIREFIGHTING** A firefighter spraying water on a fire stood on the middle rung of a ladder. The smoke lessened, so she moved up 3 rungs. It got too hot, so she backed down 5 rungs. Later, she went up 7 rungs and stayed until the fire was out. Then, she climbed the remaining 4 rungs and went into the building. How many rungs does the ladder have?

21. **MONEY** Hugo withdrew some money from his bank account. He spent one third of the money for gasoline. Then he spent half of what was left for a haircut. He bought lunch for $6.55. When he got home, he had $13.45 left. How much did he withdraw from the bank?

Solve each equation. Then check your solution.

22. \(5n + 6 = -4\)
23. \(7 + 3c = -11\)
24. \(15 = 4a - 5\)
25. \(-63 = 7g - 14\)
26. \(-\frac{c}{3} + 5 = 7\)
27. \(\frac{y}{5} + 9 = 6\)
28. \(3 - \frac{a}{7} = -2\)
29. \(-9 - \frac{p}{4} = 5\)
30. \(\frac{1}{8} - 6 = -12\)
31. \(\frac{m}{5} + 6 = 31\)
32. \(\frac{17 - s}{4} = -10\)
33. \(-\frac{3j - (-4)}{-6} = 12\)
34. \(-3d - 1.2 = 0.9\)
35. \(-2.5r - 32.7 = 74.1\)
36. \(-0.6 + (-4a) = -1.4\)
37. \(\frac{p}{7} - 0.5 = 1.3\)
38. \(3.5x + 5 - 1.5x = 8\)
39. \(\frac{9z + 4}{5} - 8 = 5.4\)

40. If \(3a - 9 = 6\), what is the value of \(5a + 2\)?
41. If \(2x + 1 = 5\), what is the value of \(3x - 4\)?

Write an equation and solve each problem.

42. Six less than two thirds of a number is negative ten. Find the number.
43. Twenty-nine is thirteen added to four times a number. What is the number?
44. Find three consecutive odd integers whose sum is 51.
45. Find three consecutive even integers whose sum is \(-30\).
46. Find four consecutive integers whose sum is 94.
47. Find four consecutive odd integers whose sum is 8.

48. **BUSINESS** Adele Jones is on a business trip and plans to rent a subcompact car from Speedy Rent-A-Car. Her company has given her a budget of $60 per day for car rental. What is the maximum distance Ms. Jones can drive in one day and still stay within her budget?
49. **GEOMETRY** The measures of the three sides of a triangle are consecutive even integers. The perimeter of the triangle is 54 centimeters. What are the lengths of the sides of the triangle?

50. **MOUNTAIN CLIMBING** A general rule for those climbing more than 7000 feet above sea level is to allow a total of \( \frac{a - 7000}{2000} + 2 \) weeks of camping during the ascension. In this expression, \( a \) represents the altitude in feet. If a group of mountain climbers have allowed for 9 weeks of camping in their schedule, how high can they climb without worrying about altitude sickness?

51. **SHOE SIZE** For Exercises 51 and 52, use the following information.
If \( \ell \) represents the length of a person’s foot in inches, the expression \( 2\ell - 12 \) can be used to estimate his or her shoe size.

52. What is the approximate length of the foot of a person who wears size 8?

53. **SALES** Trever Goetz is a salesperson who is paid a monthly salary of $500 plus a 2% commission on sales. How much must Mr. Goetz sell to earn $2000 this month?

54. **GEOMETRY** A rectangle is cut from the corner of a 10-inch by 10-inch of paper. The area of the remaining piece of paper is \( \frac{4}{5} \) of the area of the original piece of paper. If the width of the rectangle removed from the paper is 4 inches, what is the length of the rectangle?

55. **CRITICAL THINKING** Determine whether the following statement is sometimes, always, or never true.

The sum of two consecutive even numbers equals the sum of two consecutive odd numbers.

56. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can equations be used to estimate the age of an animal?

Include the following in your answer:

- an explanation of how to solve the equation representing the age of the alligator, and
- an estimate of the age of the alligator.

57. Which equation represents the following problem?

Fifteen minus three times a number equals negative twenty-two. Find the number.

- A. \( 3n - 15 = -22 \)
- B. \( 15 - 3n = -22 \)
- C. \( 3(15 - n) = -22 \)
- D. \( 3(n - 15) = -22 \)

58. Which equation has a solution of \(-5\)?

- A. \( 2a - 6 = 4 \)
- B. \( 3a + 7 = 8 \)
- C. \( \frac{3a - 7}{4} = 2 \)
- D. \( \frac{3}{5}a + 19 = 16 \)
EQUATION SOLVER  You can use a graphing calculator to solve equations that are rewritten as expressions that equal zero.

**Step 1** Write the equation so that one side is equal to 0.

**Step 2** On a TI-83 Plus, press [MATH] and choose 0, for solve.

**Step 3** Enter the equation after 0=. Use [ALPHA] to enter the variables. Press [ENTER].

**Step 4** Press [ALPHA] [SOLVE] to reveal the solution. Use the ▲ key to begin entering a new equation.

Use a graphing calculator to solve each equation.

59. \(0 = 11y + 33\)
60. \(w + \frac{2}{5} - 4 = 0\)
61. \(6 = -12 + \frac{b}{-7}\)
62. \(\frac{p - (-5)}{-2} = 6\)
63. \(0.7 = \frac{r - 0.8}{6}\)
64. \(4.91 + 7.2t = 38.75\)

**Maintain Your Skills**

**Mixed Review** Solve each equation. Then check your solution.  (Lesson 3-3)

65. \(-7t = 91\)
66. \(\frac{r}{15} = -8\)
67. \(-\frac{2}{3}h = -1\frac{1}{2}\)

**TRANSPORTATION** For Exercises 68 and 69, use the following information.
In the year 2000, there were 18 more models of sport utility vehicles than there were in the year 1990. There were 47 models of sport utility vehicles in 2000.  (Lesson 3-2)

68. Write an addition equation to represent the situation.
69. How many models of sport utility vehicles were there in 1990?

Find the odds of each outcome if you spin the spinner at the right.  (Lesson 2-6)

70. spinning a number divisible by 3
71. spinning a number equal to or greater than 5
72. spinning a number less than 7

Find each quotient.  (Lesson 2-4)

73. \(-\frac{6}{7} \div 3\)
74. \(\frac{2}{8}\)
75. \(-\frac{3a + 16}{4}\)
76. \(\frac{15t - 25}{-5}\)

Use the Distributive Property to find each product.  (Lesson 1-5)

77. \(17 \cdot 9\)
78. \(13(101)\)
79. \(16(1\frac{1}{4})\)
80. \(18(2\frac{1}{9})\)

Write an algebraic expression for each verbal expression.  (Lesson 1-1)

81. the product of 5 and \(m\) plus half of \(n\)
82. the quantity 3 plus \(b\) divided by \(y\)
83. the sum of 3 times \(a\) and the square of \(b\)

**Getting Ready for the Next Lesson**

**PREREQUISITE SKILL** Simplify each expression.  (To review simplifying expressions, see Lesson 1-5.)

84. \(5d - 2d\)
85. \(11m - 5m\)
86. \(8t + 6t\)
87. \(7g - 15g\)
88. \(-9f + 6f\)
89. \(-3m + (-7m)\)
Solving Equations with the Variable on Each Side

**What You'll Learn**
- Solve equations with the variable on each side.
- Solve equations involving grouping symbols.

**Vocabulary**
- *identity*

**How can an equation be used to determine when two populations are equal?**

In 1995, there were 18 million Internet users in North America. Of this total, 12 million were male, and 6 million were female. During the next five years, the number of male Internet users on average increased 7.6 million per year, and the number of female Internet users increased 8 million per year. If this trend continues, the following expressions represent the number of male and female Internet users $x$ years after 1995.

Male Internet Users: $12 + 7.6x$
Female Internet Users: $6 + 8x$

The equation $12 + 7.6x = 6 + 8x$ represents the time at which the number of male and female Internet users are equal. Notice that this equation has the variable $x$ on each side.

**VARIABLES ON EACH SIDE** Many equations contain variables on each side. To solve these types of equations, first use the Addition or Subtraction Property of Equality to write an equivalent equation that has all of the variables on one side.

**Example 1 Solve an Equation with Variables on Each Side**

Solve $-2 + 10p = 8p - 1$. Then check your solution.

$-2 + 10p = 8p - 1$ \hspace{1cm} \text{Original equation}

$-2 + 10p - 8p = 8p - 1 - 8p$ \hspace{1cm} \text{Subtract 8p from each side.}

$-2 + 2p = -1$ \hspace{1cm} \text{Simplify.}

$-2 + 2p + 2 = -1 + 2$ \hspace{1cm} \text{Add 2 to each side.}

$2p = 1$ \hspace{1cm} \text{Simplify.}

$p = \frac{1}{2}$ or 0.5 \hspace{1cm} \text{Divide each side by 2.}

**CHECK** $-2 + 10p = 8p - 1$ \hspace{1cm} \text{Original equation}

$-2 + 10(0.5) = 8(0.5) - 1$ \hspace{1cm} \text{Substitute 0.5 for p.}

$-2 + 5 = 4 - 1$ \hspace{1cm} \text{Multiply.}

$3 = 3$ \hspace{1cm} \text{The solution is} \frac{1}{2} \text{ or 0.5.}
GROUPING SYMBOLS  When solving equations that contain grouping symbols, first use the Distributive Property to remove the grouping symbols.

Example 2  Solve an Equation with Grouping Symbols

Solve $4(2r - 8) = \frac{1}{7}(49r + 70)$. Then check your solution.

\[
\begin{align*}
4(2r - 8) & = \frac{1}{7}(49r + 70) & \text{Original equation} \\
8r - 32 & = 7r + 10 & \text{Distributive Property} \\
8r - 32 - 7r & = 7r + 10 - 7r & \text{Subtract } 7r \text{ from each side.} \\
r - 32 & = 10 & \text{Simplify.} \\
r - 32 + 32 & = 10 + 32 & \text{Add } 32 \text{ to each side.} \\
r & = 42 & \text{Simplify.}
\end{align*}
\]

CHECK

\[
\begin{align*}
4(2r - 8) & = \frac{1}{7}(49r + 70) & \text{Original equation} \\
4[2(42) - 8] & = \frac{1}{7}[49(42) + 70] & \text{Substitute } 42 \text{ for } r. \\
4(84 - 8) & = \frac{1}{7}(2058 + 70) & \text{Multiply.} \\
4(76) & = \frac{1}{7}(2128) & \text{Add and subtract.} \\
304 & = 304 & \checkmark
\end{align*}
\]

The solution is 42.

Some equations with the variable on each side may have no solution. That is, there is no value of the variable that will result in a true equation.

Example 3  No Solutions

Solve $2m + 5 = 5(m - 7) - 3m$.

\[
\begin{align*}
2m + 5 & = 5(m - 7) - 3m & \text{Original equation} \\
2m + 5 & = 5m - 35 - 3m & \text{Distributive Property} \\
2m + 5 & = 2m - 35 & \text{Simplify.} \\
2m + 5 - 2m & = 2m - 35 - 2m & \text{Subtract } 2m \text{ from each side.} \\
5 & = -35 & \text{This statement is false.}
\end{align*}
\]

Since $5 = -35$ is a false statement, this equation has no solution.

An equation that is true for every value of the variable is called an identity.

Example 4  An Identity

Solve $3(r + 1) - 5 = 3r - 2$.

\[
\begin{align*}
3(r + 1) - 5 & = 3r - 2 & \text{Original equation} \\
3r + 3 - 5 & = 3r - 2 & \text{Distributive Property} \\
3r - 2 & = 3r - 2 & \text{Reflexive Property of Equality}
\end{align*}
\]

Since the expressions on each side of the equation are the same, this equation is an identity. The statement $3(r + 1) - 5 = 3r - 2$ is true for all values of $r$. 
Lesson 3-5
Solving Equations with the Variable on Each Side

1. Determine whether each solution is correct. If the solution is not correct, find the error and give the correct solution.

   a. \[2(g + 5) = 22\]
      \[2g + 5 = 22\]
      \[2g + 5 - 5 = 22 - 5\]
      \[2g = 17\]
      \[\frac{2g}{2} = \frac{17}{2}\]
      \[g = 8.5\]

   b. \[5d = 2d - 18\]
      \[5d - 2d = 2d - 18 - 2d\]
      \[-6z + 3 = 7z - 6z\]
      \[3d = -18\]
      \[\frac{3d}{3} = \frac{-18}{3}\]
      \[d = -6\]

   c. \[-6z + 13 = 7z\]

Standardized Test Practice

Example 5 Use Substitution to Solve an Equation

Multiple-Choice Test Item

Solve \[2(b - 3) + 5 = 3(b - 1)\].

A \[-2\]  B \[2\]  C \[-3\]  D \[3\]

Read the Test Item
You are asked to solve an equation.

Solve the Test Item
You can solve the equation or substitute each value into the equation and see if it makes the equation true. We will solve by substitution.

A \[2(b - 3) + 5 = 3(b - 1)\]

B \[2(b - 3) + 5 = 3(b - 1)\]

\[2(-2 - 3) + 5 \neq 3(-2 - 1)\]

\[2(-5) + 5 \neq 3(-3)\]

\[-10 + 5 \neq -9\]

\[-5 \neq -9\]

Since the value 2 results in a true statement, you do not need to check -3 and 3. The answer is B.

Check for Understanding

Concept Check

1. Determine whether each solution is correct. If the solution is not correct, find the error and give the correct solution.

   a. \[2(g + 5) = 22\]
      \[2g + 5 = 22\]
      \[2g + 5 - 5 = 22 - 5\]
      \[2g = 17\]
      \[\frac{2g}{2} = \frac{17}{2}\]
      \[g = 8.5\]

   b. \[5d = 2d - 18\]
      \[5d - 2d = 2d - 18 - 2d\]
      \[-6z + 3 = 7z - 6z\]
      \[3d = -18\]
      \[\frac{3d}{3} = \frac{-18}{3}\]
      \[d = -6\]

   c. \[-6z + 13 = 7z\]
2. Explain how to determine whether an equation is an identity.
3. OPEN ENDED Find a counterexample to the statement all equations have a solution.

**Guided Practice**

4. Justify each step.

\[ 6n + 7 = 8n - 13 \]

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<td>e.</td>
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<td>f.</td>
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Solve each equation. Then check your solution.

5. \[ 20c + 5 = 5c + 65 \]
6. \[ \frac{3}{8} - \frac{1}{4}t = \frac{1}{2}t - \frac{3}{4} \]
7. \[ 3(a - 5) = -6 \]
8. \[ 7 - 3r = r - 4(2 + r) \]
9. \[ 6 = 3 + 5(d - 2) \]
10. \[ \frac{c + 1}{8} = \frac{c}{4} \]
11. \[ 5h - 7 = 5(h - 2) + 3 \]
12. \[ 5.4w + 8.2 = 9.8w - 2.8 \]

**Standardized Test Practice**

13. Solve \( 75 - 9t = 5(-4 + 2t) \).

(A) -5  (B) -4  (C) 4  (D) 5

**Practice and Apply**

**Homework Help**

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<tr>
<th>For Exercises</th>
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**Extra Practice**

See page 826.

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Solve each equation. Then check your solution.

14. \[ \frac{3m - 2}{5} = \frac{7}{10} \]
15. \[ v + 9 = 7v + 9 \]

16. \[ 3 - 4q = 10q + 10 \]
17. \[ 3k - 5 = 7k - 21 \]
18. \[ 5t - 9 = -3t + 7 \]
19. \[ 8s + 9 = 7s + 6 \]
20. \[ \frac{3}{4}n + 16 = 2 - \frac{1}{8}n \]
21. \[ \frac{1}{4} - \frac{2}{3}y = \frac{3}{4} - \frac{1}{3}y \]
22. \[ 8 = 4(3c + 5) \]
23. \[ 7(m - 3) = 7 \]
24. \[ 6(r + 2) - 4 = -10 \]
25. \[ 5 - \frac{1}{2}(x - 6) = 4 \]
26. \[ 4(2a - 1) = -10(a - 5) \]
27. \[ 4(f - 2) = 4f \]
28. \[ 3(1 + d) - 5 = 3d - 2 \]
29. \[ 2(w - 3) + 5 = 3(w - 1) \]
30. \( \frac{3}{2}y - y = 4 + \frac{1}{2}y \)
31. \( 3 + \frac{2}{5}b = 11 - \frac{2}{5}b \)
32. \( \frac{1}{4}(7 + 3g) = -\frac{8}{8} \)
33. \( \frac{1}{6}(a - 4) = \frac{1}{3}(2a + 4) \)
34. \( 28 - 2.2x = 11.6x + 262.6 \)
35. \( 1.03p - 4 = -2.15p + 8.72 \)
36. \( 18 - 3.8t = 7.36 - 1.9t \)
37. \( 13.7v - 6.5 = -2.3v + 8.3 \)
38. \( 2[s + 3(s - 1)] = 18 \)
39. \( -3(2n - 5) = 0.5(-12n + 30) \)
40. One half of a number increased by 16 is four less than two thirds of the number. Find the number.
41. The sum of one half of a number and 6 equals one third of the number. What is the number?
42. **NUMBER THEORY** Twice the greater of two consecutive odd integers is 13 less than three times the lesser number. Find the integers.
43. **NUMBER THEORY** Three times the greatest of three consecutive even integers exceeds twice the least by 38. What are the integers?
44. **HEALTH** When exercising, a person’s pulse rate should not exceed a certain limit, which depends on his or her age. This maximum rate is represented by the expression \( 0.8(220 - a) \), where \( a \) is age in years. Find the age of a person whose maximum pulse is 152.
45. **HARDWARE** Traditionally, nails are given names such as 2-penny, 3-penny, and so on. These names describe the lengths of the nails. What is the name of a nail that is \( 2\frac{1}{2} \) inches long?
46. **TECHNOLOGY** About 4.9 million households had one brand of personal computers in 2001. The use of these computers grew at an average rate of 0.275 million households a year. In 2001, about 2.5 million households used another type of computer. The use of these computers grew at an average rate of 0.7 million households a year. How long will it take for the two types of computers to be in the same number of households?
47. **GEOMETRY** The rectangle and square shown below have the same perimeter. Find the dimensions of each figure.

\[
\begin{align*}
\text{Rectangle: } & 3x + 1 \\
\text{Square: } & 3x
\end{align*}
\]
48. **ENERGY** Use the information on energy at the left. The amount of energy \( E \) in BTUs needed to raise the temperature of water is represented by the equation \( E = w(t_f - t_o) \). In this equation, \( w \) represents the weight of the water in pounds, \( t_f \) represents the final temperature in degrees Fahrenheit, and \( t_o \) represents the original temperature in degrees Fahrenheit. A 50-gallon water heater is 60% efficient. If 10 cubic feet of natural gas are used to raise the temperature of water with the original temperature of 50°F, what is the final temperature of the water? (One gallon of water weighs about 8 pounds.)
49. **CRITICAL THINKING** Write an equation that has one or more grouping symbols, the variable on each side of the equals sign, and a solution of \(-2\).
50. **Writing in Math** Answer the question that was posed at the beginning of the lesson.

   How can an equation be used to determine when two populations are equal?
   Include the following in your answer:
   - a list of the steps needed to solve the equation,
   - the year when the number of female Internet users will equal the number of male Internet users according to the model, and
   - an explanation of why this method can be used to predict future events.

51. Solve $8x - 3 = 5(2x + 1)$.
   - A 4
   - B 2
   - C -2
   - D -4

52. Solve $5n + 4 = 7(n + 1) - 2n$.
   - A 0
   - B -1
   - C no solution
   - D all numbers

**Standardized Test Practice**

53. Solve $2v - 6 = 14$.
54. Solve $\frac{x - 3}{7} = -2$.
55. $5 - 9w = 23$

**Maintain Your Skills**

**Mixed Review** Solve each equation. Then check your solution. **(Lesson 3-4)**

53. $\frac{2}{9}v - 6 = 14$
54. $\frac{x - 3}{7} = -2$
55. $5 - 9w = 23$

**HEALTH** For Exercises 56 and 57, use the following information.
Ebony burns 4.5 Calories per minute pushing a lawn mower. **(Lesson 3-3)**

56. Write a multiplication equation representing the number of Calories $C$ burned if Ebony pushes the lawn mower for $m$ minutes.
57. How long will it take Ebony to burn 150 Calories mowing the lawn?

Use each set of data to make a line plot. **(Lesson 2-5)**

58. 13, 15, 11, 15, 16, 17, 12, 12, 13, 15, 16, 15
59. 22, 25, 19, 21, 22, 24, 22, 25, 28, 21, 24, 22

Find each sum or difference. **(Lesson 2-2)**

60. $-10 + (-17)$
61. $-12 - (-8)$
62. $6 - 14$

Write a counterexample for each statement. **(Lesson 1-7)**

63. If the sum of two numbers is even, then both addends are even.
64. If you are baking cookies, you will need chocolate chips.

Evaluate each expression when $a = 5$, $b = 8$, $c = 7$, $x = 2$, and $y = 1$. **(Lesson 1-2)**

65. $\frac{3x^2}{b + c}$
66. $x(a + 2b) - y$
67. $5(x + 2y) - 4a$

**Getting Ready for the Next Lesson**

**PREREQUISITE SKILL** Simplify each fraction. **(To review simplifying fractions, see pages 798 and 799.)**

68. $\frac{12}{15}$
69. $\frac{28}{49}$
70. $\frac{36}{60}$
71. $\frac{8}{120}$
72. $\frac{108}{9}$
73. $\frac{28}{42}$
74. $\frac{16}{40}$
75. $\frac{19}{57}$
**What You’ll Learn**

- Determine whether two ratios form a proportion.
- Solve proportions.

**Vocabulary**

- ratio
- proportion
- extremes
- means
- rate
- scale

**How are ratios used in recipes?**

The ingredients in the recipe will make 4 servings of honey frozen yogurt. Keri can use ratios and equations to find the amount of each ingredient needed to make enough yogurt for her club meeting.

**RATIOS AND PROPORTIONS**

A **ratio** is a comparison of two numbers by division. The ratio of \( x \) to \( y \) can be expressed in the following ways.

\[
\frac{x}{y} \quad x:y \quad \frac{x}{y}
\]

Ratios are often expressed in simplest form. For example, the recipe above states that for 4 servings you need 2 cups of milk. The ratio of servings to milk may be written as 4 to 2, 4:2, or \( \frac{4}{2} \). Written in simplest form, the ratio of servings to milk can be written as 2 to 1, 2:1, or \( \frac{2}{1} \).

Suppose you wanted to double the recipe to have 8 servings. The amount of milk required would be 4 cups. The ratio of servings to milk is \( \frac{8}{4} \). When this ratio is simplified, the ratio is \( \frac{2}{1} \). Notice that this ratio is equal to the original ratio.

\[
\begin{align*}
\frac{4}{2} &= \frac{2}{1} \\
\frac{8}{4} &= \frac{2}{1}
\end{align*}
\]

An equation stating that two ratios are equal is called a **proportion**. So, we can state that \( \frac{4}{2} = \frac{8}{4} \) is a proportion.

**Example 1**

**Determine Whether Ratios Form a Proportion**

Determine whether the ratios \( \frac{4}{5} \) and \( \frac{24}{30} \) form a proportion.

\[
\begin{align*}
\frac{4}{5} &= \frac{4}{5} \\
\frac{24}{30} &= \frac{4}{5}
\end{align*}
\]

The ratios are equal. Therefore, they form a proportion.
Another way to determine whether two ratios form a proportion is to use cross products. If the cross products are equal, then the ratios form a proportion.

**Example 2**  
*Use Cross Products*

Use cross products to determine whether each pair of ratios form a proportion.

a. \( \frac{0.4}{0.8} = \frac{0.7}{1.4} \)

Write the equation.

\( 0.4(1.4) \neq 0.8(0.7) \)  
Find the cross products.

0.56 = 0.56  
Simplify.

The cross products are equal, so \( \frac{0.4}{0.8} = \frac{0.7}{1.4} \). Since the ratios are equal, they form a proportion.

b. \( \frac{6}{8} \neq \frac{24}{28} \)

Write the equation.

\( 6(28) \neq 8(24) \)  
Find the cross products.

168 \neq 192  
Simplify.

The cross products are not equal, so \( \frac{6}{8} \neq \frac{24}{28} \). The ratios do not form a proportion.

In the proportion \( \frac{0.4}{0.8} = \frac{0.7}{1.4} \) above, 0.4 and 1.4 are called the **extremes**, and 0.8 and 0.7 are called the **means**.

**Key Concept**  
*Means–Extremes Property of Proportion*

- **Words**  
  In a proportion, the product of the extremes is equal to the product of the means.

- **Symbols**  
  If \( \frac{a}{b} = \frac{c}{d} \), then \( ad = bc \).

- **Examples**  
  Since \( \frac{2}{4} = \frac{1}{2} \), \( 2(2) = 4(1) \) or \( 4 = 4 \).

**SOLVE PROPORTIONS**  
You can write proportions that involve a variable. To solve the proportion, use cross products and the techniques used to solve other equations.

**Example 3**  
*Solve a Proportion*

Solve the proportion \( \frac{n}{15} = \frac{24}{16} \).

\( \frac{n}{15} = \frac{24}{16} \)  
Original equation

16\( n \) = 15(24)  
Find the cross products.

16\( n \) = 360  
Simplify.

\( \frac{16n}{16} = \frac{360}{16} \)  
Divide each side by 16.

\( n = 22.5 \)  
Simplify.
The ratio of two measurements having different units of measure is called a **rate**. For example, a price of $1.99 per dozen eggs, a speed of 55 miles per hour, and a salary of $30,000 per year are all rates. Proportions are often used to solve problems involving rates.

---

**Example 4**  
**Use Rates**

**BICYCLING** Trent goes on a 30-mile bike ride every Saturday. He rides the distance in 4 hours. At this rate, how far can he ride in 6 hours?

**Explore** Let \( m \) represent the number of miles Trent can ride in 6 hours.

**Plan** Write a proportion for the problem.

\[
\frac{\text{miles}}{\text{hours}} = \frac{30}{4} = \frac{m}{6} \quad \leftrightarrow \quad \text{miles} \quad \leftrightarrow \quad \text{hours}
\]

**Solve**

\[
\frac{30}{4} = \frac{m}{6} \quad \text{Original proportion}
\]

\[30(6) = 4(m) \quad \text{Find the cross products.}\]

\[180 = 4m \quad \text{Simplify.}\]

\[\frac{180}{4} = \frac{4m}{4} \quad \text{Divide each side by 4.}\]

\[45 = m \quad \text{Simplify.}\]

**Examine** If Trent rides 30 miles in 4 hours, he rides 7.5 miles in 1 hour. So, in 6 hours, Trent can ride \( 6 \times 7.5 \) or 45 miles. The answer is correct.

Since the rates are equal, they form a proportion. So, Trent can ride 45 miles in 6 hours.

---

A ratio or rate called a **scale** is used when making a model or drawing of something that is too large or too small to be conveniently drawn at actual size. The scale compares the model to the actual size of the object using a proportion. Maps and blueprints are two commonly used scale drawings.

**Example 5**  
**Use a Scale Drawing**

**CRATER LAKE** The scale of a map for Crater Lake National Park is 2 inches = 9 miles. The distance between Discovery Point and Phantom Ship Overlook on the map is about \( 1\frac{3}{4} \) inches. What is the distance between these two places?

Let \( d \) represent the actual distance.

\[
\frac{\text{scale}}{\text{actual}} = \frac{2}{9} = \frac{1\frac{3}{4}}{d} \quad \leftrightarrow \quad \text{scale} \quad \leftrightarrow \quad \text{actual}
\]

\[2(d) = 9\left(1\frac{3}{4}\right) \quad \text{Find the cross products.}\]

\[2d = \frac{63}{4} \quad \text{Simplify.}\]

\[2d \div 2 = \frac{63}{4} \div 2 \quad \text{Divide each side by 2.}\]

\[d = \frac{63}{8} \quad \text{or} \quad 7\frac{7}{8} \quad \text{Simplify.}\]

The actual distance is about \( 7\frac{7}{8} \) miles.
Check for Understanding

Concept Check
1. **OPEN ENDED** Find an example of ratios used in advertisements.
2. **Explain** the difference between a ratio and a proportion.
3. **Describe** how to solve a proportion if one of the ratios contains a variable.

Guided Practice
Use cross products to determine whether each pair of ratios form a proportion. Write yes or no.

- 4. \( \frac{4}{12} \) \( \frac{11}{33} \)
- 5. \( \frac{16}{21} \) \( \frac{8}{9} \)
- 6. \( \frac{21}{3.5} \) \( \frac{0.5}{0.7} \)

Solve each proportion. If necessary, round to the nearest hundredth.

- 7. \( \frac{3}{4} = \frac{6}{x} \)
- 8. \( \frac{a}{45} = \frac{5}{15} \)
- 9. \( \frac{0.6}{1.1} = \frac{n}{8.47} \)

Application
10. **TRAVEL** The Lehmans’ minivan requires 5 gallons of gasoline to travel 120 miles. How much gasoline will they need for a 350-mile trip?

Practice and Apply

Use cross products to determine whether each pair of ratios form a proportion. Write yes or no.

- 11. \( \frac{3}{4} \) \( \frac{21}{14} \)
- 12. \( \frac{8}{9} \) \( \frac{12}{18} \)
- 13. \( \frac{23}{3.4} \) \( \frac{3.0}{3.6} \)
- 14. \( \frac{4.2}{5.6} \) \( \frac{1.68}{2.24} \)
- 15. \( \frac{21.1}{14.4} \) \( \frac{1.1}{1.2} \)
- 16. \( \frac{5}{2} \) \( \frac{4}{1.6} \)

**SPORTS** For Exercises 17 and 18, use the graph at the right.

17. Write a ratio of the number of gold medals won to the total number of medals won for each country.

18. Do any two of the ratios you wrote for Exercise 17 form a proportion? If so, explain the real-world meaning of the proportion.

Solve each proportion. If necessary, round to the nearest hundredth.

- 19. \( \frac{4}{x} = \frac{2}{10} \)
- 20. \( \frac{1}{y} = \frac{3}{15} \)
- 21. \( \frac{6}{5} = \frac{x}{15} \)
- 22. \( \frac{20}{n} = \frac{n}{28} \)
- 23. \( \frac{6}{8} = \frac{7}{a} \)
- 24. \( \frac{16}{7} = \frac{9}{b} \)
- 25. \( \frac{1}{0.19} = \frac{12}{n} \)
- 26. \( \frac{2}{0.21} = \frac{8}{n} \)
- 27. \( \frac{2.405}{3.67} = \frac{5}{1.88} \)
- 28. \( \frac{7}{1.066} = \frac{z}{9.65} \)
- 29. \( \frac{6}{14} = \frac{7}{x - 3} \)
- 30. \( \frac{5}{3} = \frac{6}{x + 2} \)
31. **WORK** Seth earns $152 in 4 days. At that rate, how many days will it take him to earn $532?

32. **DRIVING** Lanette drove 248 miles in 4 hours. At that rate, how long will it take her to drive an additional 93 miles?

33. **BLUEPRINTS** A blueprint for a house states that 2.5 inches equals 10 feet. If the length of a wall is 12 feet, how long is the wall in the blueprint?

34. **MODELS** A collector’s model racecar is scaled so that 1 inch on the model equals $6\frac{1}{4}$ feet on the actual car. If the model is $\frac{2}{3}$ inch high, how high is the actual car?

35. **PETS** A research study shows that three out of every twenty pet owners got their pet from a breeder. Of the 122 animals cared for by a veterinarian, how many would you expect to have been bought from a breeder?

36. **CRITICAL THINKING** Consider the proportion $\frac{a}{b} = \frac{3}{5}$. What is the value of $\frac{\text{?}}{b}$? (Hint: Choose different values of $a$, $b$, and $c$ for which the proportion is true and evaluate the expression.)

37. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

**How are ratios used in recipes?**

Include the following in your answer:

- an explanation of how to use a proportion to determine how much honey is needed if you use 3 eggs, and
- a description of how to alter the recipe to get 5 servings.

38. Which ratio is not equal to $\frac{9}{12}$?

- A. $\frac{18}{24}$
- B. $\frac{3}{4}$
- C. $\frac{15}{20}$
- D. $\frac{18}{27}$

39. In the figure at the right, $\frac{x}{y} = 2:3$ and $\frac{y}{z} = 3:5$. If $x = 10$, find the value of $z$.

- A. 15
- B. 20
- C. 25
- D. 30

**Mixed Review**

Solve each equation. Then check your solution. *(Lessons 3-4 and 3-5)*

40. $8y - 10 = -3y + 2$
41. $17 + 2n = 21 + 2n$
42. $-7(d - 3) = -4$

43. $5 - 9w = 23$
44. $\frac{m}{-5} + 6 = 31$
45. $\frac{z - 7}{5} = -3$

Find each product. *(Lesson 2-3)*

46. $(-7)(-6)$
47. $\left(-\frac{8}{9}\right)\left(-\frac{9}{8}\right)$
48. $\left(\frac{3}{7}\right)\left(\frac{3}{7}\right)$
49. $(-0.075)(-5.5)$

Find each absolute value. *(Lesson 2-1)*

50. $| -33 |$
51. $| 77 |$
52. $| 2.5 |$
53. $| -0.85 |$

54. Sketch a reasonable graph for the temperature in the following statement. *In August, you enter a hot house and turn on the air conditioner.* *(Lesson 1-8)*

**Getting Ready for the Next Lesson**

**PREREQUISITE SKILL** Find each percent. *(To review percents, see pages 802 and 803.)*

55. Eighteen is what percent of 60?
56. What percent of 14 is 4.34?
57. Six is what percent of 15?
58. What percent of 2 is 8?
Percent of Change

Vocabulary
- percent of change
- percent of increase
- percent of decrease

What You’ll Learn
- Find percents of increase and decrease.
- Solve problems involving percents of change.

How can percents describe growth over time?
Phone companies began using area codes in 1947. The graph shows the number of area codes in use in different years. The growth in the number of area codes can be described by using a percent of change.

PERCENT OF CHANGE
When an increase or decrease is expressed as a percent, the percent is called the percent of change. If the new number is greater than the original number, the percent of change is a percent of increase. If the new number is less than the original, the percent of change is a percent of decrease.

Example 1 Find Percent of Change
State whether each percent of change is a percent of increase or a percent of decrease. Then find each percent of change.

a. original: 25
   new: 28
Find the amount of change. Since the new amount is greater than the original, the percent of change is a percent of increase.
   \[28 - 25 = 3\]
Find the percent using the original number, 25, as the base.
   \[
   \text{original amount} \rightarrow \frac{3}{25} = \frac{r}{100}
   \]
   \[3(100) = 25(r)\]
   \[300 = 25r\]
   \[r = \frac{300}{25}\]
   \[r = 12\]
   The percent of increase is 12%.

b. original: 30
   new: 12
   The percent of change is a percent of decrease because the new amount is less than the original. Find the change.
   \[30 - 12 = 18\]
Find the percent using the original number, 30, as the base.
   \[
   \text{original amount} \rightarrow \frac{18}{30} = \frac{r}{100}
   \]
   \[18(100) = 30(r)\]
   \[1800 = 30r\]
   \[r = \frac{1800}{30}\]
   \[r = 60\]
   The percent of decrease is 60%.

Look Back
To review the percent proportion, see page 834.
Lesson 3-7  Percent of Change

**Example 2  Find the Missing Value**

**FOOTBALL** The field used by the National Football League (NFL) is 120 yards long. The length of the field used by the Canadian Football League (CFL) is 25% longer than the one used by the NFL. What is the length of the field used by the CFL?

Let \( \ell \) = the length of the CFL field. Since 25% is a percent of increase, the length of the NFL field is less than the length of the CFL field. Therefore, \( \ell - 120 \) represents the amount of change.

\[
\text{change} \rightarrow \frac{\ell - 120}{120} = \frac{25}{100} \quad \text{Percent proportion}
\]

Find the cross products.

\[
(\ell - 120)(100) = 120(25)
\]

Distributive Property

\[
100\ell - 12,000 = 3000 \quad \text{Add 12,000 to each side.}
\]

Simplify.

\[
\frac{100\ell}{100} = \frac{15,000}{100}
\]

Divide each side by 100.

\[
\ell = 150
\]

Simplify.

The length of the field used by the CFL is 150 yards.

**SOLVE PROBLEMS** Two applications of percent of change are sales tax and discounts. Sales tax is a tax that is added to the cost of the item. It is an example of a percent of increase. Discount is the amount by which the regular price of an item is reduced. It is an example of a percent of decrease.

**Example 3  Find Amount After Sales Tax**

**SALES TAX** A concert ticket costs $45. If the sales tax is 6.25%, what is the total price of the ticket?

The tax is 6.25% of the price of the ticket.

\[
6.25\% \text{ of } $45 = 0.0625 \times 45 \quad 6.25\% = 0.0625
\]

Use a calculator.

Round $2.8125 to $2.81.

Add this amount to the original price.

$45.00 + $2.81 = $47.81

The total price of the ticket is $47.81.

**Example 4  Find Amount After Discount**

**DISCOUNT** A sweater is on sale for 35% off the original price. If the original price of the sweater is $38, what is the discounted price?

The discount is 35% of the original price.

\[
35\% \text{ of } $38 = 0.35 \times 38 \quad 35\% = 0.35
\]

Use a calculator.

Subtract $13.30 from the original price.

$38.00 - $13.30 = $24.70

The discounted price of the sweater is $24.70.

---

**Football**

On November 12, 1892, the Allegheny Athletic Association paid William “Pudge” Heffelfinger $500 to play football. This game is considered the start of professional football.

Source: World Book Encyclopedia

www.algebra1.com/extra_examples
1. **Compare and contrast** percent of increase and percent of decrease.

2. **OPEN ENDED** Give a counterexample to the statement *The percent of change must always be less than 100%*.

3. **FIND THE ERROR** Laura and Cory are writing proportions to find the percent of change if the original number is 20 and the new number is 30.

   \[
   \frac{\text{Amount of change}}{\text{Original}} = \frac{10}{20} = \frac{r}{100}
   \]

   Who is correct? Explain your reasoning.

**Guided Practice**

State whether each percent of change is a percent of increase or a percent of decrease. Then find each percent of change. Round to the nearest whole percent.

4. original: 72
   new: 36
5. original: 45
   new: 50
6. original: 14
   new: 16
7. original: 150
   new: 120

Find the total price of each item.

8. software: $39.50
   sales tax: 6.5%
9. compact disc: $15.99
   sales tax: 5.75%

Find the discounted price of each item.

10. jeans: $45.00
    discount: 25%
11. book: $19.95
    discount: 33%

**Application**

**EDUCATION** For Exercises 12 and 13, use the following information.

According to the Census Bureau, the average income of a person with a bachelor’s degree is $40,478, and the average income of a person with a high school diploma is $22,895.

12. Write an equation that could be used to find the percent of increase in average income for a person with a high school diploma to average income for a person with a bachelor’s degree.

13. What is the percent of increase?

**Practice and Apply**

State whether each percent of change is a percent of increase or a percent of decrease. Then find each percent of change. Round to the nearest whole percent.

14. original: 50
    new: 70
15. original: 25
    new: 18
16. original: 66
    new: 30
17. original: 58
    new: 152
18. original: 13.7
    new: 40.2
19. original: 15.6
    new: 11.4
20. original: 132
    new: 150
21. original: 85
    new: 90
22. original: 32.5
    new: 30
23. original: 9.8
    new: 12.1
24. original: 40
    new: 32.5
25. original: 25
    new: 21.5
26. **THEME PARKS**  In 1990, 253 million people visited theme parks in the United States. In 2000, the number of visitors increased to 317 million people. What was the percent of increase?

27. **MILITARY**  In 1987, the United States had 2 million active-duty military personnel. By 2000, there were only 1.4 million active-duty military personnel. What was the percent of decrease?

28. The percent of increase is 16%. If the new number is 522, find the original number.

29. **FOOD**  In order for a food to be marked “reduced fat,” it must have at least 25% less fat than the same full-fat food. If one ounce of reduced fat chips has 6 grams of fat, what is the least amount of fat in one ounce of regular chips?


Find the total price of each item.

31. umbrella: $14.00  
tax: 5.5%

32. backpack: $35.00  
tax: 7%

33. candle: $7.50  
tax: 5.75%

34. hat: $18.50  
tax: 6.25%

35. clock radio: $39.99  
tax: 6.75%

36. sandals: $29.99  
tax: 5.75%

Find the discounted price of each item.

37. shirt: $45.00  
discount: 40%

38. socks: $6.00  
discount: 20%

39. watch: $37.55  
discount: 35%

40. gloves: $24.25  
discount: 33%

41. suit: $175.95  
discount: 45%

42. coat: $79.99  
discount: 30%

Find the final price of each item.

43. lamp: $120.00  
discount: 20%  
tax: 6%

44. dress: $70.00  
discount: 30%  
tax: 7%

45. camera: $58.00  
discount: 25%  
tax: 6.5%

**POPULATION**  For Exercises 46 and 47, use the following table.

<table>
<thead>
<tr>
<th>Country</th>
<th>1997 Population (billions)</th>
<th>Projected Percent of Increase for 2050</th>
</tr>
</thead>
<tbody>
<tr>
<td>China</td>
<td>1.24</td>
<td>22.6%</td>
</tr>
<tr>
<td>India</td>
<td>0.97</td>
<td>57.8%</td>
</tr>
<tr>
<td>United States</td>
<td>0.27</td>
<td>44.4%</td>
</tr>
</tbody>
</table>

Source: USA TODAY

46. What are the projected 2050 populations for each country in the table?

47. Which of these three countries is projected to be the most populous in 2050?

48. **RESEARCH**  Use the Internet or other reference to find the tuition for the last several years at a college of your choice. Find the percent of change for the tuition during these years. Predict the tuition for the year you plan to graduate from high school.

49. **CRITICAL THINKING**  Are the following expressions sometimes, always, or never equal? Explain your reasoning.

\[ x\% \text{ of } y \]  

\[ y\% \text{ of } x \]
50. **Writing in Math**  
Answer the question that was posed at the beginning of the lesson.

How can percents describe growth over time?
Include the following in your answer:
- the percent of increase in the number of area codes from 1996 to 1999, and
- an explanation of why knowing a percent of change can be more informative than knowing how much the quantity changed.

51. The number of students at Franklin High School increased from 840 to 910 over a 5-year period. Which proportion represents the percent of change?
   - **A** \( \frac{70}{910} = \frac{r}{100} \)
   - **B** \( \frac{70}{840} = \frac{r}{100} \)
   - **C** \( \frac{r}{910} = \frac{70}{100} \)
   - **D** \( \frac{r}{840} = \frac{70}{100} \)

52. The list price of a television is $249.00. If it is on sale for 30% off the list price, what is the sale price of the television?
   - **A** $74.70
   - **B** $149.40
   - **C** $174.30
   - **D** $219.00

### Maintain Your Skills

#### Mixed Review

**Solve each proportion.** (Lesson 3-6)

53. \( \frac{a}{45} = \frac{3}{15} \)
54. \( \frac{2}{3} = \frac{8}{d} \)
55. \( \frac{5.22}{13.92} = \frac{t}{48} \)

**Solve each equation. Then check your solution.** (Lesson 3-5)

56. \( 6n + 3 = -3 \)
57. \( 7 + 5c = -23 \)
58. \( 18 = 4a - 2 \)

**Find each quotient.** (Lesson 2-4)

59. \( \frac{2}{3} \div 4 \)
60. \( -\frac{4}{5} \div \frac{2}{3} \)
61. \( -\frac{1}{9} \div \left(-\frac{3}{4}\right) \)

**State whether each equation is true or false for the value of the variable given.** (Lesson 1-3)

62. \( a^2 + 5 = 17 - a \), \( a = 3 \)
63. \( 2v^2 + v = 65 \), \( v = 5 \)
64. \( 8y - y^2 = y + 10 \), \( y = 4 \)
65. \( 16p - p = 15p \), \( p = 2.5 \)

### Getting Ready for the Next Lesson

**PREREQUISITE SKILL**  
Solve each equation. Then check your solution.  
*(To review solving equations, see Lesson 3-5.)*

66. \( -43 - 3t = 2 - 6t \)
67. \( 7y + 7 = 3y - 5 \)
68. \( 7(d - 3) - 2 = 5 \)
69. \( 6(p + 3) = 4(p - 1) \)
70. \( -5 = 4 - 2(a - 5) \)
71. \( 8x - 4 = -10x + 50 \)

### Practice Quiz 2

**Lessons 3-4 through 3-7**

**Solve each equation. Then check your solution.** (Lessons 3-4 and 3-5)

1. \(-3x - 7 = 18\)
2. \(5 = \frac{m - 5}{4}\)
3. \(4h + 5 = 11\)
4. \(5d - 6 = 3d + 9\)
5. \(7 + 2(w + 1) = 2w + 9\)
6. \(-8(4 + 9r) = 7(-2 - 11r)\)

**Solve each proportion.** (Lesson 3-6)

7. \(\frac{2}{10} = \frac{1}{a}\)
8. \(\frac{3}{5} = \frac{24}{x}\)
9. \(\frac{y}{4} = \frac{y + 5}{8}\)

10. **Postage**  
In 1975, the cost of a first-class stamp was 10¢. In 2001, the cost of a first-class stamp became 34¢. What is the percent of increase in the price of a stamp?  
*(Lesson 3-7)*
Recall that you can solve percent problems using two different methods. With either method, it is helpful to use “clue” words such as *is* and *of*. In the sentence method, *is* means equals and *of* means multiply. With the proportion method, the “clue” words indicate where to place the numbers in the proportion.

<table>
<thead>
<tr>
<th>Sentence Method</th>
<th>Proportion Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>15% of 40 is what number?</td>
<td>15% of 40 is what number?</td>
</tr>
<tr>
<td>0.15 \cdot 40 = ?</td>
<td>[ \frac{(is) \ P}{(of) \ B} = \frac{R(\text{percent})}{100} \rightarrow \frac{P}{40} = \frac{15}{100} ]</td>
</tr>
</tbody>
</table>

You can use the proportion method to solve percent of change problems. In this case, use the proportion $\frac{\text{difference}}{\text{original}} = \frac{\%}{100}$. When reading a percent of change problem, or any other word problem, look for the important numerical information.

**Example** In chemistry class, Kishi heated 20 milliliters of water. She let the water boil for 10 minutes. Afterward, only 17 milliliters of water remained, due to evaporation. What is the percent of decrease in the amount of water?

\[
\frac{\text{difference}}{\text{original}} = \frac{\%}{100} \rightarrow \frac{20 - 17}{20} = \frac{r}{100} \\
3 \cdot 20 = 20(r) \quad \text{Percent proportion} \\
3(100) = 20(r) \quad \text{Simplify.} \\
300 = 20r \quad \text{Find the cross products.} \\
300 = 20r \quad \text{Simplify.} \\
15 = r \quad \text{Divide each side by 20.} \\
\]

There was a 15% decrease in the amount of water.

**Reading to Learn**

Give the original number and the amount of change. Then write and solve a percent proportion.

1. Monsa needed to lose weight for wrestling. At the start of the season, he weighed 166 pounds. By the end of the season, he weighed 158 pounds. What is the percent of decrease in Monsa’s weight?

2. On Carla’s last Algebra test, she scored 94 points out of 100. On her first Algebra test, she scored 75 points out of 100. What is the percent of increase in her score?

3. In a catalog distribution center, workers processed an average of 12 orders per hour. After a reward incentive was offered, workers averaged 18 orders per hour. What is the percent of increase in production?
SOLVE FOR VARIABLES

Some equations such as the one above contain more than one variable. At times, you will need to solve these equations for one of the variables.

**Example 1** Solve an Equation for a Specific Variable

Solve $3x - 4y = 7$ for $y$.

Original equation

Solve $3x - 4y = 7$ for $y$.

Subtract $3x$ from each side.

Simplify.

Divide each side by $-4$.

Simplify.

The value of $y$ is $\frac{3x - 7}{4}$.
Lesson 3-8
Solving Equations and Formulas

Example 2  Solve an Equation for a Specific Variable

Solve $2m - t = sm + 5$ for $m$.

$2m - t = sm + 5$  Original equation
$2m - t - sm = sm + 5 - sm$  Subtract $sm$ from each side.
$2m - t - sm + t = 5 + t$  Add $t$ to each side.
$2m - sm = 5 + t$  Simplify.
$m(2 - s) = 5 + t$  Use the Distributive Property.
$m(2 - s) = \frac{5 + t}{2 - s}$  Divide each side by $2 - s$.
$m = \frac{5 + t}{2 - s}$  Simplify.

The value of $m$ is $\frac{5 + t}{2 - s}$. Since division by 0 is undefined, $2 - s \neq 0$ or $s \neq 2$.

USE FORMULAS  Many real-world problems require the use of formulas. Sometimes solving a formula for a specific variable will help you solve the problem.

Example 3  Use a Formula to Solve Problems

WEATHER  Use the information about the Kansas City hailstorm at the left. The formula for the circumference of a circle is $C = 2\pi r$, where $C$ represents circumference and $r$ represents radius.

a.  Solve the formula for $r$.

$C = 2\pi r$  Formula for circumference
$\frac{C}{2\pi} = \frac{2\pi r}{2\pi}$  Divide each side by $2\pi$.
$\frac{C}{2\pi} = r$  Simplify.

b.  Find the radius of one of the largest hailstones that fell on Kansas City in 1898.

$\frac{C}{2\pi} = r$  Formula for radius
$\frac{9.5}{2\pi} = r$  $C = 9.5$
$1.5 \approx r$  The largest hailstones had a radius of about 1.5 inches.

When using formulas, you may want to use dimensional analysis. Dimensional analysis is the process of carrying units throughout a computation.

Example 4  Use Dimensional Analysis

PHYSICAL SCIENCE  The formula $s = \frac{1}{2}at^2$ represents the distance $s$ that a free-falling object will fall near a planet or the moon in a given time $t$. In the formula, $a$ represents the acceleration due to gravity.

a.  Solve the formula for $a$.

$s = \frac{1}{2}at^2$  Original formula
$\frac{2}{t^2}(s) = \frac{2}{t^2}\left(\frac{1}{2}at^2\right)$  Multiply each side by $\frac{2}{t^2}$.
$\frac{2s}{t^2} = a$  Simplify.

www.algebra1.com/extra_examples
b. A free-falling object near the moon drops 20.5 meters in 5 seconds. What is the value of \(a\) for the moon?

\[
a = \frac{2s}{t^2}
\]

Formula for \(a\)

\[
a = \frac{2(20.5m)}{(5s)^2}
\]

\(s = 20.5m\) and \(t = 5s\).

\[
a = \frac{1.64m}{s^2} \text{ or } 1.64 \text{ m/s}^2
\]

Use a calculator.

The acceleration due to gravity on the moon is 1.64 meters per second squared.

---

**Check for Understanding**

**Concept Check**

1. List the steps you would use to solve \(ax - y = az + w\) for \(a\).

2. Describe the possible values of \(t\) if \(s = \frac{r}{t - 2}\).

3. **OPEN ENDED** Write a formula for \(A\), the area of a geometric figure such as a triangle or rectangle. Then solve the formula for a variable other than \(A\).

**Guided Practice**

Solve each equation or formula for the variable specified.

4. \(-3x + b = 6x\), for \(x\)

5. \(-5a + y = -54\), for \(a\)

6. \(4z + b = 2z + c\), for \(z\)

7. \(\frac{y + a}{3} = c\), for \(y\)

8. \(p = a(b + c)\), for \(a\)

9. \(mw - t = 2w + 5\), for \(w\)

**Application** **GEOMETRY** For Exercises 10–12, use the formula for the area of a triangle.

10. Find the area of a triangle with a base of 18 feet and a height of 7 feet.

11. Solve the formula for \(h\).

12. What is the height of a triangle with area of 28 square feet and base of 8 feet?

---

**Practice and Apply**

Solve each equation or formula for the variable specified.

13. \(5g + h = g\), for \(g\)

14. \(8t - r = 12t\), for \(t\)

15. \(y = mx + b\), for \(m\)

16. \(v = r + at\), for \(a\)

17. \(3y + z = am - 4y\), for \(y\)

18. \(9a - 2b = c + 4a\), for \(a\)

19. \(km + 5x = 6y\), for \(m\)

20. \(4b - 5 = -t\), for \(b\)

21. \(\frac{3ax - n}{5} = -4\), for \(x\)

22. \(\frac{5x + y}{a} = 2\), for \(a\)

23. \(\frac{by + 2}{3} = c\), for \(y\)

24. \(\frac{6c - t}{7} = b\), for \(c\)

25. \(c = \frac{3}{4}y + b\), for \(y\)

26. \(\frac{3}{5}m + a = b\), for \(m\)

27. \(S = \frac{n}{2}(A + t)\), for \(A\)

28. \(p(t + 1) = -2\), for \(t\)

29. \(at + b = ar - c\), for \(a\)

30. \(2g - m = 5 - gh\), for \(g\)

---

**Homework Help**

For Exercises 13–30: See Examples 1, 2

For Exercises 31–41: See Examples 3, 4

**Extra Practice**

See page 827.
Write an equation and solve for the variable specified.

31. Five less than a number \( t \) equals another number \( r \) plus six. Solve for \( t \).

32. Five minus twice a number \( p \) equals six times another number \( q \) plus one. Solve for \( p \).

33. Five eighths of a number \( x \) is three more than one half of another number \( y \). Solve for \( y \).

**GEOMETRY**  For Exercises 34 and 35, use the formula for the area of a trapezoid.

34. Solve the formula for \( h \).

35. What is the height of a trapezoid with an area of 60 square meters and bases of 8 meters and 12 meters?

**WORK**  For Exercises 36 and 37, use the following information.

The formula \( s = \frac{w - 10e}{m} \) is often used by placement services to find keyboarding speeds. In the formula, \( s \) represents the speed in words per minute, \( w \) represents the number of words typed, \( e \) represents the number of errors, and \( m \) represents the number of minutes typed.

36. Solve the formula for \( e \).

37. If Miguel typed 410 words in 5 minutes and received a keyboard speed of 76 words per minute, how many errors did he make?

**FLOORING**  For Exercises 38 and 39, use the following information.

The formula \( P = \frac{1.2W}{H^2} \) represents the amount of pressure exerted on the floor by the heel of a shoe. In this formula, \( P \) represents the pressure in pounds per square inch, \( W \) represents the weight of a person wearing the shoe in pounds, and \( H \) is the width of the heel of the shoe in inches.

38. Solve the formula for \( W \).

39. Find the weight of the person if the heel is 3 inches wide and the pressure exerted is 30 pounds per square inch.

**ROCKETRY**  In the book *October Sky*, high school students were experimenting with different rocket designs. One formula they used was \( R = \frac{S + F + P}{S + P} \), which relates the mass ratio \( R \) of a rocket to the mass of the structure \( S \), the mass of the fuel \( F \), and the mass of the payload \( P \). The students needed to determine how much fuel to load in the rocket. How much fuel should be loaded in a rocket whose basic structure and payload each have a mass of 900 grams, if the mass ratio is to be 6?

**PACKAGING**  The Yummy Ice Cream Company wants to package ice cream in cylindrical containers that have a volume of 5453 cubic centimeters. The marketing department decides the diameter of the base of the containers should be 20 centimeters. How tall should the containers be?

(Hint: \( V = \pi r^2 h \))
42. **CRITICAL THINKING** Write a formula for the area of the arrow.

43. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

**How are equations used to design roller coasters?**
Include the following in your answer:
- a list of steps you could use to solve the equation for \(h\), and
- the height of the second hill of the roller coaster.

44. If \(2x + y = 5\), what is the value of \(4x\)?
- **A** \(10 - y\)
- **B** \(10 - 2y\)
- **C** \(5 - \frac{y}{2}\)
- **D** \(\frac{10 - y}{2}\)

45. What is the area of the triangle?
- **A** \(23 \text{ m}^2\)
- **B** \(28 \text{ m}^2\)
- **C** \(56 \text{ m}^2\)
- **D** \(112 \text{ m}^2\)

---

**Maintain Your Skills**

**Mixed Review** Find the discounted price of each item. *(Lesson 3-7)*

46. camera: $85.00 discount: 20%
47. scarf: $15.00 discount: 35%
48. television: $299.00 discount: 15%

Solve each proportion. *(Lesson 3-6)*

49. \(\frac{2}{9} = \frac{5}{a}\)
50. \(\frac{15}{32} = \frac{1}{8}\)
51. \(\frac{x + 1}{8} = \frac{3}{4}\)

Write the numbers in each set in order from least to greatest. *(Lesson 2-7)*

52. \(\frac{1}{4}, \sqrt{\frac{1}{4}}, 0.5, 0.2\)
53. \(\sqrt{5}, 3, \frac{2}{3}, 1.1\)

Find each sum or difference. *(Lesson 2-2)*

54. \(2.18 + (-5.62)\)
55. \(-\frac{1}{2} - \left(-\frac{3}{4}\right)\)
56. \(-\frac{2}{3} - \frac{2}{5}\)

Name the property illustrated by each statement. *(Lesson 1-4)*

57. \(mnp = 1mnp\)
58. If \(6 = 9 - 3\), then \(9 - 3 = 6\).
59. \(32 + 21 = 32 + 21\)
60. \(8 + (3 + 9) = 8 + 12\)

---

**Getting Ready for the Next Lesson**

**PREREQUISITE SKILL** Use the Distributive Property to rewrite each expression without parentheses. *(To review the Distributive Property, see Lesson 1-5.)*

61. \(6(2 - t)\)
62. \((5 + 2m)3\)
63. \(-7(3a + b)\)
64. \(\frac{2}{3}(6h - 9)\)
65. \(-\frac{3}{5}(15 - 5t)\)
66. \(0.25(6p + 12)\)
Vocabulary
- weighted average
- mixture problem
- uniform motion problem

How are scores calculated in a figure skating competition?

In individual figure skating competitions, the score for the long program is worth twice the score for the short program. Suppose Olympic gold medal winner Ilia Kulik scores 5.5 in the short program and 5.8 in the long program at a competition. His final score is determined using a weighted average.

\[
\frac{5.5(1) + 5.8(2)}{1 + 2} = \frac{5.5 + 11.6}{3} = \frac{17.1}{3} = 5.7
\]

His final score would be 5.7.

MIXTURE PROBLEMS Ilia Kulik’s average score is an example of a weighted average. The weighted average \( M \) of a set of data is the sum of the product of the number of units and the value per unit divided by the sum of the number of units.

Mixture problems are problems in which two or more parts are combined into a whole. They are solved using weighted averages.

Example 1 Solve a Mixture Problem with Prices

TRAIL MIX Assorted dried fruit sells for $5.50 per pound. How many pounds of mixed nuts selling for $4.75 per pound should be mixed with 10 pounds of dried fruit to obtain a trail mix that sells for $4.95 per pound?

Let \( w \) = the number of pounds of mixed nuts in the mixture. Make a table.

<table>
<thead>
<tr>
<th>Units (lb)</th>
<th>Price per Unit (lb)</th>
<th>Total Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dried Fruit</td>
<td>10</td>
<td>$5.50</td>
</tr>
<tr>
<td>Mixed Nuts</td>
<td>( w )</td>
<td>$4.75</td>
</tr>
<tr>
<td>Trail Mix</td>
<td>( 10 + w )</td>
<td>$4.95</td>
</tr>
</tbody>
</table>

Price of dried fruit plus price of nuts equals price of trail mix.

\[
5.50(10) + 4.75w = 4.95(10 + w)
\]

Original equation

\[
55.00 + 4.75w = 49.50 + 4.95w
\]

Distributive Property

\[
55.00 + 4.75w - 4.75w = 49.50 + 4.95w - 4.75w
\]

Subtract 4.75w from each side.

\[
55.00 = 49.50 + 0.20w
\]

Simplify.

\[
55.00 - 49.50 = 49.50 + 0.20w - 49.50
\]

Subtract 49.50 from each side.

\[
5.50 = 0.20w
\]

Simplify.

\[
\frac{5.50}{0.20} = \frac{0.20w}{0.20}
\]

Divide each side by 0.20.

\[
27.5 = w
\]

Simplify.

27.5 pounds of nuts should be mixed with 10 pounds of dried fruit.
Sometimes mixture problems are expressed in terms of percents.

**Example 2.** Solve a Mixture Problem with Percents

**SCIENCE** A chemistry experiment calls for a 30% solution of copper sulfate. Kendra has 40 milliliters of 25% solution. How many milliliters of 60% solution should she add to obtain the required 30% solution?

Let $x$ be the amount of 60% solution to be added. Make a table.

<table>
<thead>
<tr>
<th>Solution (mL)</th>
<th>Amount of Copper Sulfate</th>
</tr>
</thead>
<tbody>
<tr>
<td>25% Solution</td>
<td>40</td>
</tr>
<tr>
<td>60% Solution</td>
<td>$x$</td>
</tr>
<tr>
<td>30% Solution</td>
<td>$40 + x$</td>
</tr>
</tbody>
</table>

Write and solve an equation using the information in the table.

\[
0.25(40) + 0.60x = 0.30(40 + x)
\]

**Original equation**

\[
10 + 0.60x = 12 + 0.30x
\]

**Distributive Property**

\[
10 + 0.60x - 0.30x = 12 + 0.30x - 0.30x
\]

**Subtract 0.30x from each side.**

\[
10 + 0.30x = 12
\]

**Simplify.**

\[
10 + 0.30x - 10 = 12 - 10
\]

**Subtract 10 from each side.**

\[
0.30x = 2
\]

**Simplify.**

\[
\frac{0.30x}{0.30} = \frac{2}{0.30}
\]

**Divide each side by 0.30.**

\[
x = 6.67
\]

**Simplify.**

Kendra should add 6.67 milliliters of the 60% solution to the 40 milliliters of the 25% solution.

**UNIFORM MOTION PROBLEMS** Motion problems are another application of weighted averages. **Uniform motion problems** are problems where an object moves at a certain speed, or rate. The formula $d = rt$ is used to solve these problems. In the formula, $d$ represents distance, $r$ represents rate, and $t$ represents time.

**Example 3.** Solve for Average Speed

**TRAVEL** On Alberto’s drive to his aunt’s house, the traffic was light, and he drove the 45-mile trip in one hour. However, the return trip took him two hours. What was his average speed for the round trip?

To find the average speed for each leg of the trip, rewrite $d = rt$ as $r = \frac{d}{t}$.

**Going**

\[r = \frac{d}{t} = \frac{45 \text{ miles}}{1 \text{ hour}} = 45 \text{ miles per hour}\]

**Returning**

\[r = \frac{d}{t} = \frac{45 \text{ miles}}{2 \text{ hours}} = 22.5 \text{ miles per hour}\]
You may think that the average speed of the trip would be \( \frac{45 + 22.5}{2} \) or 33.75 miles per hour. However, Alberto did not drive at these speeds for equal amounts of time. You must find the weighted average for the trip.

**Round Trip**

\[
M = \frac{45(1) + 22.5(2)}{1 + 2} \quad \text{Definition of weighted average}
\]

\[
= \frac{90}{3} \quad \text{or} \quad 30 \quad \text{Simplify.}
\]

Alberto’s average speed was 30 miles per hour.

Sometimes a table is useful in solving uniform motion problems.

**Example 4** Solve a Problem Involving Speeds of Two Vehicles

**SAFETY** Use the information about sirens at the left. A car and an emergency vehicle are heading toward each other. The car is traveling at a speed of 30 miles per hour or about 44 feet per second. The emergency vehicle is traveling at a speed of 50 miles per hour or about 74 feet per second. If the vehicles are 1000 feet apart and the conditions are ideal, in how many seconds will the driver of the car first hear the siren?

Draw a diagram. The driver can hear the siren when the total distance traveled by the two vehicles equals 1000 feet or 560 feet.

Let \( t \) = the number of seconds until the driver can hear the siren. Make a table of the information.

<table>
<thead>
<tr>
<th></th>
<th>( r )</th>
<th>( t )</th>
<th>( d = rt )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car</td>
<td>44</td>
<td>( t )</td>
<td>44t</td>
</tr>
<tr>
<td>Emergency Squad</td>
<td>74</td>
<td>( t )</td>
<td>74t</td>
</tr>
</tbody>
</table>

Write an equation.

\[
\frac{\text{distance traveled by car}}{44t} \quad \text{plus} \quad \frac{\text{distance traveled by emergency vehicle}}{74t} \quad \text{equals} \quad \frac{560 \text{ feet}}{560}
\]

Solve the equation.

\[
44t + 74t = 560 \quad \text{Original equation}
\]

\[
118t = 560 \quad \text{Simplify.}
\]

\[
\frac{118t}{118} = \frac{560}{118}
\]

\[
t \approx 4.75 \quad \text{Round to the nearest hundredth.}
\]

The driver of the car will hear the siren in about 4.75 seconds.
Check for Understanding

**Concept Check**

1. **OPEN ENDED** Give a real-world example of a weighted average.
2. **Write** the formula used to solve uniform motion problems and tell what each letter represents.
3. **Make a table** that can be used to solve the following problem.
   
   Lakeisha has $2.55 in dimes and quarters. She has 8 more dimes than quarters. How many quarters does she have?

**Guided Practice**

**FOOD** For Exercises 4–7, use the following information.
How many quarts of pure orange juice should Michael add to a 10% orange drink to create 6 quarts of a 40% orange juice mixture? Let \( p \) represent the number of quarts of pure orange juice he should add to the orange drink.

4. Copy and complete the table representing the problem.

<table>
<thead>
<tr>
<th>Quarts</th>
<th>Amount of Orange Juice</th>
</tr>
</thead>
<tbody>
<tr>
<td>10% Juice</td>
<td>6 − ( p )</td>
</tr>
<tr>
<td>100% Juice</td>
<td>( p )</td>
</tr>
<tr>
<td>40% Juice</td>
<td></td>
</tr>
</tbody>
</table>

5. Write an equation to represent the problem.
6. How much pure orange juice should Michael use?
7. How much 10% juice should Michael use?

**BUSINESS** The Nut Shoppe sells walnuts for $4.00 a pound and cashews for $7.00 a pound. How many pounds of cashews should be mixed with 10 pounds of walnuts to obtain a mixture that sells for $5.50 a pound?

**GRADES** Many schools base a student’s grade point average, or GPA, on the student’s grade and the class credit rating. Brittany’s grade card for this semester is shown. Find Brittany’s GPA if a grade of A equals 4 and a B equals 3.

**CYCLING** Two cyclists begin traveling in the same direction on the same bike path. One travels at 20 miles per hour, and the other travels at 14 miles per hour. When will the cyclists be 15 miles apart?

Practice and Apply

**Homework Help**

<table>
<thead>
<tr>
<th>For Exercises</th>
<th>See Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>11–18, 22–25, 27–29, 33</td>
<td>1, 2</td>
</tr>
<tr>
<td>19–21, 26, 30–32, 34</td>
<td>3, 4</td>
</tr>
</tbody>
</table>

**Extra Practice**

See page 828.

**BUSINESS** For Exercises 11–14, use the following information.
Cookies Inc. sells peanut butter cookies for $6.50 per dozen and chocolate chip cookies for $9.00 per dozen. Yesterday, they sold 85 dozen more peanut butter cookies than chocolate chip cookies. The total sales for both types of cookies were $4055.50. Let \( p \) represent the number of dozens of peanut butter cookies sold.

11. Copy and complete the table representing the problem.

<table>
<thead>
<tr>
<th>Number of Dozens</th>
<th>Price per Dozen</th>
<th>Total Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peanut Butter Cookies</td>
<td>( p )</td>
<td></td>
</tr>
<tr>
<td>Chocolate Chip Cookies</td>
<td>( p − 85 )</td>
<td></td>
</tr>
</tbody>
</table>

12. Write an equation to represent the problem.
13. How many dozen peanut butter cookies were sold?
14. How many dozen chocolate chip cookies were sold?
**METALS** For Exercises 15–18, use the following information.
In 2000, the international price of gold was $270 per ounce, and the international price of silver was $5 per ounce. Suppose gold and silver were mixed to obtain 15 ounces of an alloy worth $164 per ounce. Let \( g \) represent the amount of gold used in the alloy.

15. Copy and complete the table representing the problem.

<table>
<thead>
<tr>
<th>Number of Ounces</th>
<th>Price per Ounce</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold</td>
<td>( g )</td>
<td></td>
</tr>
<tr>
<td>Silver</td>
<td>( 15 - g )</td>
<td></td>
</tr>
<tr>
<td>Alloy</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

16. Write an equation to represent the problem.
17. How much gold was used in the alloy?
18. How much silver was used in the alloy?

**TRAVEL** For Exercises 19–21, use the following information.
Two trains leave Pittsburgh at the same time, one traveling east and the other traveling west. The eastbound train travels at 40 miles per hour, and the westbound train travels at 30 miles per hour. Let \( t \) represent the amount of time since their departure.

19. Copy and complete the table representing the situation.

<table>
<thead>
<tr>
<th>Eastbound Train</th>
<th>Westbound Train</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r )</td>
<td>( t )</td>
</tr>
</tbody>
</table>

20. Write an equation that could be used to determine when the trains will be 245 miles apart.
21. In how many hours will the trains be 245 miles apart?

22. **FUND-RAISING** The Madison High School marching band sold gift wrap. The gift wrap in solid colors sold for $4.00 per roll, and the print gift wrap sold for $6.00 per roll. The total number of rolls sold was 480, and the total amount of money collected was $2340. How many rolls of each kind of gift wrap were sold?

23. **COFFEE** Charley Baroni owns a specialty coffee store. He wants to create a special mix using two coffees, one priced at $6.40 per pound and the other priced at $7.28 per pound. How many pounds of the $7.28 coffee should he mix with 9 pounds of the $6.40 coffee to sell the mixture for $6.95 per pound?

24. **FOOD** Refer to the graphic at the right. How much whipping cream and 2% milk should be mixed to obtain 35 gallons of milk with 4% butterfat?

25. **METALS** An alloy of metals is 25% copper. Another alloy is 50% copper. How much of each alloy should be used to make 1000 grams of an alloy that is 45% copper?

26. **TRAVEL** An airplane flies 1000 miles due east in 2 hours and 1000 miles due south in 3 hours. What is the average speed of the airplane?
27. **SCIENCE**  Hector is performing a chemistry experiment that requires 140 milliliters of a 30% copper sulfate solution. He has a 25% copper sulfate solution and a 60% copper sulfate solution. How many milliliters of each solution should he mix to obtain the needed solution?

28. **CAR MAINTENANCE**  One type of antifreeze is 40% glycol, and another type of antifreeze is 60% glycol. How much of each kind should be used to make 100 gallons of antifreeze that is 48% glycol?

29. **GRADES**  In Ms. Martinez’s science class, a test is worth three times as much as a quiz. If a student has test grades of 85 and 92 and quiz grades of 82, 75, and 95, what is the student’s average grade?

30. **RESCUE**  A fishing trawler has radioed the Coast Guard for a helicopter to pick up an injured crew member. At the time of the emergency message, the trawler is 660 kilometers from the helicopter and heading toward it. The average speed of the trawler is 30 kilometers per hour, and the average speed of the helicopter is 300 kilometers per hour. How long will it take the helicopter to reach the trawler?

31. **ANIMALS**  A cheetah is 300 feet from its prey. It starts to sprint toward its prey at 90 feet per second. At the same time, the prey starts to sprint at 70 feet per second. When will the cheetah catch its prey?

32. **TRACK AND FIELD**  A sprinter has a bad start, and his opponent is able to start 1 second before him. If the sprinter averages 8.2 meters per second and his opponent averages 8 meters per second, will he be able to catch his opponent before the end of the 200-meter race? Explain.

33. **CAR MAINTENANCE**  A car radiator has a capacity of 16 quarts and is filled with a 25% antifreeze solution. How much must be drained off and replaced with pure antifreeze to obtain a 40% antifreeze solution?

34. **TRAVEL**  An express train travels 80 kilometers per hour from Ironton to Wildwood. A local train, traveling at 48 kilometers per hour, takes 2 hours longer for the same trip. How far apart are Ironton and Wildwood?

35. **FOOTBALL**  NFL quarterbacks are rated for their passing performance by a type of weighted average as described in the formula below.

\[
R = \left[ 50 + \frac{200(C + A)}{24} + \frac{8000(T + A)}{24} - 10,000(I + A) + 100(Y + A) \right] / 24
\]

In this formula,
- \(R\) represents the rating,
- \(C\) represents number of completions,
- \(A\) represents the number of passing attempts,
- \(T\) represents the number of touchdown passes,
- \(I\) represents the number of interceptions, and
- \(Y\) represents the number of yards gained by passing.

In the 2000 season, Daunte Culpepper had 297 completions, 474 passing attempts, 33 touchdown passes, 16 interceptions, and 3937 passing yards. What was his rating for that year?


36. **CRITICAL THINKING**  Write a mixture problem for the equation

\[1.00x + 0.28(40) = 0.40(x + 40)\]
37. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are scores calculated in a figure skating competition?

Include the following in your answer:
• an explanation of how a weighted average can be used to find a skating score, and
• a demonstration of how to find the weighted average of a skater who received a 4.9 in the short program and a 5.2 in the long program.

38. Eula Jones is investing $6000 in two accounts, part at 4.5% and the remainder at 6%. If \( d \) represents the number of dollars invested at 4.5%, which expression represents the amount of interest earned in one year by the account paying 6%?

A. \( 0.06d \)  
B. \( 0.06(d - 6000) \)  
C. \( 0.06(d + 6000) \)  
D. \( 0.06(6000 - d) \)

39. Todd drove from Boston to Cleveland, a distance of 616 miles. His breaks, gasoline, and food stops took 2 hours. If his trip took 16 hours altogether, what was his average speed?

A. 38.5 mph  
B. 40 mph  
C. 44 mph  
D. 47.5 mph

### Maintain Your Skills

#### Mixed Review

Solve each equation for the variable specified. *(Lesson 3-8)*

40. \( 3t - 4 = 6t - s \), for \( t \)  
41. \( a + 6 = \frac{b - 1}{4} \), for \( b \)

State whether each percent of change is a percent of increase or a percent of decrease. Then find the percent of change. Round to the nearest whole percent. *(Lesson 3-7)*

42. original: 25  
   new: 14  
43. original: 35  
   new: 42  
44. original: 244  
   new: 300

45. If the probability that an event will occur is \( \frac{2}{3} \), what are the odds that the event will occur? *(Lesson 2-6)*

Simplify each expression. *(Lesson 2-3)*

46. \( (2b)(-3a) \)  
47. \( 3x(-3y) + (-6x)(-2y) \)  
48. \( 5s(-6t) + 2s(-8t) \)

Name the set of numbers graphed. *(Lesson 2-1)*

49.  
50.  

### WebQuest

**Can You Fit 100 Candles on a Cake?**

It’s time to complete your project. Use the information and data you have gathered about living to be 100 to prepare a portfolio or Web page. Be sure to include graphs and/or tables in the presentation.

[www.algebra1.com/webquest](http://www.algebra1.com/webquest)
Finding a Weighted Average

You can use a computer spreadsheet program to calculate weighted averages. A spreadsheet allows you to make calculations and print almost anything that can be organized in a table.

The basic unit in a spreadsheet is called a cell. A cell may contain numbers, words, or a formula. Each cell is named by the column and row that describe its location. For example, cell B4 is in column B, row 4.

Example

Greta Norris manages the Java Roaster Coffee Shop. She has entered the price per pound and the number of pounds sold in October for each type of coffee in a spreadsheet. What was the average price per pound of coffee sold?

The spreadsheet shows the formula that will calculate the weighted average. The formula multiplies the price of each product by its volume and calculates its sum for all the products. Then it divides that value by the sum of the volume for all products together. To the nearest cent, the weighted average of a pound of coffee is $11.75.

Exercises

For Exercises 1–4, use the spreadsheet of coffee prices.

1. What is the average price of a pound of coffee for the November sales shown in the table at the right?
2. How does the November weighted average change if all of the coffee prices are increased by $1.00?
3. How does the November weighted average change if all of the coffee prices are increased by 10%?
4. Find the weighted average of a pound of coffee if the shop sold 50 pounds of each type of coffee. How does the weighted average compare to the average of the per-pound coffee prices? Explain.

<table>
<thead>
<tr>
<th>Product</th>
<th>Pounds Sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hawaiian Cafe</td>
<td>56</td>
</tr>
<tr>
<td>Mocha Java</td>
<td>97</td>
</tr>
<tr>
<td>House Blend</td>
<td>124</td>
</tr>
<tr>
<td>Decaf Espresso</td>
<td>71</td>
</tr>
<tr>
<td>Breakfast Blend</td>
<td>69</td>
</tr>
<tr>
<td>Italian Roast</td>
<td>45</td>
</tr>
</tbody>
</table>
Choose the correct term to complete each sentence.

1. According to the (Addition, Multiplication) Property of Equality, if \( a = b \), then \( a + c = b + c \).

2. A (means, ratio) is a comparison of two numbers by division.

3. A rate is the ratio of two measurements with (the same, different) units of measure.

4. The first step in the four-step problem-solving plan is to (explore, solve) the problem.

5. \( 2x + 1 = 2x + 1 \) is an example of a(n) (identity, formula).

6. An equivalent equation for \( 3x + 5 = 7 \) is \( 3x = 2, 3x = 12 \).

7. If the original amount was 80 and the new amount is 90, then the percent of (decrease, increase) is 12.5%.

8. (Defining the variable, Dimensional analysis) is the process of carrying units throughout a computation.

9. The (weighted average, rate) of a set of data is the sum of the product of each number in the set and its weight divided by the sum of all the weights.

10. An example of consecutive integers is (8 and 9, 8 and 10).

---

**Lesson-by-Lesson Review**

### Writing Equations

**Concept Summary**
- Variables are used to represent unknowns when writing equations.
- Formulas given in sentence form can be written as algebraic equations.

**Example**

Translate the following sentence into an equation.

*The sum of \( x \) and \( y \) equals 2 plus two times the product of \( x \) and \( y \).*

\[
\frac{ x + y }{ x + y } = \frac{ 2 }{ 2 } + \frac{ \text{two times the product of } x \text{ and } y }{ 2xy }
\]

The equation is \( x + y = 2 + 2xy \).
Exercises  Translate each sentence into an equation.  
11. Three times a number \( n \) decreased by 21 is 57.  
12. Four minus three times \( z \) is equal to \( z \) decreased by 2.  
13. The sum of the square of \( a \) and the cube of \( b \) is 16.  
14. Translate the equation \( 16 - 9r = r \) into a verbal sentence.  

Solving Equations by Using Addition and Subtraction  

**Concept Summary**  
- **Addition Property of Equality**  
  For any numbers \( a, b, \) and \( c \), if \( a = b \), then \( a + c = b + c \).  
- **Subtraction Property of Equality**  
  For any numbers \( a, b, \) and \( c \), if \( a = b \), then \( a - c = b - c \).  

**Example**  
Solve \( x - 13 = 45 \). Then check your solution.  

\[
x - 13 = 45 \quad \text{Original equation} \\
x - 13 + 13 = 45 + 13 \quad \text{Add 13 to each side.} \\
x = 58 \quad \text{Simplify.} \\
\]

**CHECK**  
\[
x - 13 = 45 \quad \text{Original equation} \\
58 - 13 \overset{?}{=} 45 \quad \text{Substitute 58 for } x. \\
45 = 45 \checkmark \quad \text{Simplify.} \quad \text{The solution is 58.} \\
\]

**Exercises**  
Solve each equation. Then check your solution.  
See Examples 1–4 on pages 129 and 130.  
15. \( r - 21 = -37 \)  
16. \( 14 + c = -5 \)  
17. \( 27 = 6 + p \)  
18. \( b + (-14) = 6 \)  
19. \( d - (-1.2) = -7.3 \)  
20. \( r + \left(-\frac{1}{2}\right) = -\frac{3}{4} \)  

Solving Equations by Using Multiplication and Division  

**Concept Summary**  
- **Multiplication Property of Equality**  
  For any numbers \( a, b, \) and \( c \), if \( a = b \), then \( ac = bc \).  
- **Division Property of Equality**  
  For any numbers \( a, b, \) and \( c \), with \( c \neq 0 \), if \( a = b \), then \( \frac{a}{c} = \frac{b}{c} \).  

**Example**  
Solve \( \frac{4}{9}t = -72 \).  

\[
\frac{4}{9}t = -72 \quad \text{Original equation} \\
\frac{9}{4} \left(\frac{4}{9}t\right) = \frac{9}{4} (-72) \quad \text{Multiply each side by } \frac{9}{4}. \\
t = -162 \quad \text{Simplify.} \\
\]

**CHECK**  
\[
\frac{4}{9}t = -72 \quad \text{Original equation} \\
\frac{4}{9} (-162) \overset{?}{=} -72 \quad \text{Substitute } -162 \text{ for } t. \\
-72 = -72 \checkmark \quad \text{Simplify.} \\
\]

The solution is \(-162\).  

See pages 128–134.

---

1-2 3-2 3-3
Exercises  Solve each equation. Then check your solution.
See Examples 1–3 on pages 135 and 136.

21. \(6x = -42\)  
22. \(-7w = -49\)  
23. \(\frac{3}{4}n = 30\)

24. \(-\frac{3}{5}y = -50\)  
25. \(\frac{5}{2}a = -25\)  
26. \(5 = \frac{r}{2}\)

### 3-4 Solving Multi-Step Equations

**Concept Summary**
- Multi-step equations can be solved by undoing the operations in reverse of the order of operations.

**Example**

Solve \(34 = 8 - 2t\). Then check your solution.

\[
\begin{align*}
34 &= 8 - 2t & \text{Original equation} \\
34 - 8 &= 8 - 2t - 8 & \text{Subtract 8 from each side.} \\
26 &= -2t & \text{Simplify.} \\
\frac{26}{-2} &= \frac{-2t}{-2} & \text{Divide each side by } -2. \\
-13 &= t & \text{Simplify.}
\end{align*}
\]

**CHECK**

\[
\begin{align*}
34 &= 8 - 2t & \text{Original equation} \\
34 &\neq 8 - 2(-13) & \text{Substitute } -13 \text{ for } t. \\
34 &= 34 & \text{The solution is } -13.
\end{align*}
\]

### Excerises  Solve each equation. Then check your solution.
See Examples 2–4 on page 143.

27. \(4p - 7 = 5\)  
28. \(6 = 4v + 2\)  
29. \(\frac{y}{3} + 6 = -45\)

30. \(\frac{c}{-4} - 8 = -42\)  
31. \(\frac{4d + 5}{7} = 7\)  
32. \(\frac{7n + (-1)}{8} = 8\)

### 3-5 Solving Equations with the Variable on Each Side

**Concept Summary**

**Steps for Solving Equations**

**Step 1** Use the Distributive Property to remove the grouping symbols.

**Step 2** Simplify the expressions on each side of the equals sign.

**Step 3** Use the Addition and/or Subtraction Properties of Equality to get the variables on one side of the equals sign and the numbers without variables on the other side of the equals sign.

**Step 4** Simplify the expressions on each side of the equals sign.

**Step 5** Use the Multiplication and/or Division Properties of Equalities to solve.
Example

Solve \( \frac{3}{4}q - 8 = \frac{1}{4}q + 9 \).

\[
\frac{3}{4}q - 8 = \frac{1}{4}q + 9 \\
\frac{3}{4}q - 8 - \frac{1}{4}q = \frac{1}{4}q + 9 - \frac{1}{4}q \\
\frac{1}{2}q - 8 = 9 \\
\frac{1}{2}q = 17 \\
2(\frac{1}{2}q) = 2(17) \\
q = 34
\]

The solution is 34.

Exercises
Solve each equation. Then check your solution.
See Examples 1–4 on pages 149 and 150.

33. \( n - 2 = 4 - 2n \)
34. \( 3t - 2(t + 3) = t \)
35. \( 3 - \frac{5}{6}y = 2 + \frac{1}{6}y \)
36. \( \frac{x - 2}{6} = \frac{x}{2} \)
37. \( 2(b - 3) = 3(b - 1) \)
38. \( 8.3h - 2.2 = 6.1h - 8.8 \)

3-6
See pages 155–159.

Ratios and Proportions

Concept Summary

- A ratio is a comparison of two numbers by division.
- A proportion is an equation stating that two ratios are equal.
- A proportion can be solved by finding the cross products.

If \( \frac{a}{b} = \frac{c}{d} \), then \( ad = bc \).

Example

Solve the proportion \( \frac{8}{7} = \frac{a}{1.75} \).

\[
\frac{8}{7} = \frac{a}{1.75} \\
8(1.75) = 7(a) \\
14 = 7a \\
2 = a
\]

Exercises
Solve each proportion. See Example 3 on page 156.

39. \( \frac{6}{15} = \frac{n}{45} \)
40. \( \frac{x}{11} = \frac{35}{55} \)
41. \( \frac{12}{d} = \frac{20}{15} \)
42. \( \frac{14}{20} = \frac{21}{m} \)
43. \( \frac{2}{3} = \frac{b + 5}{9} \)
44. \( \frac{6}{8} = \frac{9}{s - 4} \)
3-7

Percent of Change

Concept Summary

• The proportion \[ \frac{\text{amount of change}}{\text{original amount}} = \frac{r}{100} \] is used to find percents of change.

Example

Find the percent of change. original: $120
new: $114

First, subtract to find the amount of change.

$120 - $114 = $6

Note that since the new amount is less than the original, the percent of change will be a percent of decrease.

Then find the percent using the original number, 120, as the base.

\[ \frac{\text{change}}{\text{original amount}} \rightarrow \frac{6}{120} = \frac{r}{100} \]

Percent proportion

\[ 6(100) = 120(r) \quad \text{Find the cross products.} \]

\[ 600 = 120r \quad \text{Simplify.} \]

\[ \frac{600}{120} = \frac{120r}{120} \quad \text{Divide each side by 120.} \]

\[ 5 = r \quad \text{Simplify.} \]

The percent of decrease is 5%.

Exercises

State whether each percent of change is a percent of increase or a percent of decrease. Then find the percent of change. Round to the nearest whole percent. See Example 1 on page 160.

45. original: 40
new: 32

46. original: 50
new: 88

47. original: 35
new: 37.1

48. Find the total price of a book that costs $14.95 plus 6.25% sales tax. See Example 3 on page 161.

49. A T-shirt priced at $12.99 is on sale for 20% off. What is the discounted price? See Example 4 on page 161.

3-8

Solving Equations and Formulas

Concept Summary

• For equations with more than one variable, you can solve for one of the variables by using the same steps as solving equations with one variable.

Example

Solve \( \frac{x + y}{b} = c \) for \( x \).

\[ \frac{x + y}{b} = c \quad \text{Original equation} \]

\[ b\left(\frac{x + y}{b}\right) = b(c) \quad \text{Multiply each side by} \ b. \]

\[ x + y = bc \quad \text{Simplify.} \]

\[ x + y - y = bc - y \quad \text{Subtract} \ y \text{ from each side.} \]

\[ x = bc - y \quad \text{Simplify.} \]
Exercises
Solve each equation or formula for the variable specified.
See Examples 1 and 2 on pages 166 and 167.

50. \[ 5x = y, \text{ for } x \]
51. \[ ay - b = c, \text{ for } y \]
52. \[ yx - a = cx, \text{ for } x \]
53. \[ \frac{2y - a}{3} = \frac{a + 3b}{4}, \text{ for } y \]

Weighted Averages

Concept Summary
- The weighted average of a set of data is the sum of the product of each number in the set and its weight divided by the sum of all the weights.
- The formula \( d = rt \) is used to solve uniform motion problems.

Example

SCIENCE Mai Lin has a 35 milliliters of 30% solution of copper sulfate. How much of a 20% solution of copper sulfate should she add to obtain a 22% solution?

Let \( x \) = amount of 20% solution to be added. Make a table.

<table>
<thead>
<tr>
<th>Amount of Solution (mL)</th>
<th>Amount of Copper Sulfate</th>
</tr>
</thead>
<tbody>
<tr>
<td>30% Solution</td>
<td>35</td>
</tr>
<tr>
<td>20% Solution</td>
<td>( x )</td>
</tr>
<tr>
<td>22% Solution</td>
<td>35 + ( x )</td>
</tr>
</tbody>
</table>

\[
0.30(35) + 0.20x = 0.22(35 + x) \quad \text{Write and solve an equation.}
\]
\[
10.5 + 0.20x = 7.7 + 0.22x \quad \text{Distributive Property}
\]
\[
10.5 + 0.20x - 0.20x = 7.7 + 0.22x - 0.20x \quad \text{Subtract 0.20x from each side.}
\]
\[
10.5 = 7.7 + 0.02x \quad \text{Simplify.}
\]
\[
10.5 - 7.7 = 7.7 + 0.02x - 7.7 \quad \text{Subtract 7.7 from each side.}
\]
\[
2.8 = 0.02x \quad \text{Simplify.}
\]
\[
\frac{2.8}{0.02} = \frac{0.02x}{0.02} \quad \text{Divide each side by 0.02.}
\]
\[
140 = x \quad \text{Simplify.}
\]

Mai Lin should add 140 milliliters of the 20% solution.

Exercises

54. COFFEE Ms. Anthony wants to create a special blend using two coffees, one priced at $8.40 per pound and the other at $7.28 per pound. How many pounds of the $7.28 coffee should she mix with 9 pounds of the $8.40 coffee to sell the mixture for $7.95 per pound? See Example 1 on page 171.

55. TRAVEL Two airplanes leave Dallas at the same time and fly in opposite directions. One airplane travels 80 miles per hour faster than the other. After three hours, they are 2940 miles apart. What is the speed of each airplane? See Example 3 on pages 172 and 173.
Choose the correct term to complete each sentence.

1. The study of numbers and the relationships between them is called (consecutive, number) theory.

2. An equation that is true for (every, only one) value of the variable is called an identity.

3. When a new number is (greater than, less than) the original number, the percent of change is called a percent of increase.

Translate each sentence into an equation.

4. The sum of twice $x$ and three times $y$ is equal to thirteen.

5. Two thirds of a number is negative eight fifths.

Solve each equation. Then check your solution.

6. $-15 + k = 8$

7. $-1.2x = 7.2$

8. $k - 16 = -21$

9. $\frac{t - 7}{4} = 11$

10. $\frac{3}{4}y = -27$

11. $-12 = 7 - \frac{y}{3}$

12. $t - (-3.4) = -5.3$

13. $-3(x + 5) = 8x + 18$

14. $5a = 125$

15. $\frac{r}{5} - 3 = \frac{2r}{5} + 16$

16. $0.1r = 19$

17. $-\frac{2}{3}z = -\frac{4}{9}$

18. $-w + 11 = 4.6$

19. $2p + 1 = 5p - 11$

20. $25 - 7w = 46$

Solve each proportion.

21. $\frac{36}{t} = \frac{9}{11}$

22. $\frac{n}{4} = \frac{3.25}{52}$

23. $\frac{5}{12} = \frac{10}{x - 1}$

State whether each percent of change is a percent of increase or a percent of decrease. Then find the percent of change. Round to the nearest whole percent.

24. original: 45
   new: 9

25. original: 12
   new: 20

Solve each equation or formula for the variable specified.

26. $h = at - 0.25vt^2$, for $a$

27. $a(y + 1) = b$, for $y$

28. **SALES** Suppose the Central Perk coffee shop sells a cup of espresso for $2.00 and a cup of cappuccino for $2.50. On Friday, Destiny sold 30 more cups of cappuccino than espresso for a total of $178.50 worth of espresso and cappuccino. How many cups of each were sold?

29. **BOATING** The Yankee Clipper leaves the pier at 9:00 A.M. at 8 knots (nautical miles per hour). A half hour later, The River Rover leaves the same pier in the same direction traveling at 10 knots. At what time will The River Rover overtake The Yankee Clipper?

30. **STANDARDIZED TEST PRACTICE** If $\frac{4}{5}$ of $\frac{3}{4} = \frac{2}{5}$ of $\frac{x}{4}$, find the value of $x$.

   A 12  B 6  C 3  D $\frac{3}{2}$
1. Bailey planted a rectangular garden that is 6 feet wide by 15 feet long. What is the perimeter of the garden? (Prerequisite Skill)

A. 21 ft  B. 27 ft  C. 42 ft  D. 90 ft

2. Which of the following is true about 65 percent of 20? (Prerequisite Skill)

A. It is greater than 20.  B. It is less than 10.  C. It is less than 20.  D. Can’t tell from the information given

3. For a science project, Kelsey measured the height of a plant grown from seed. She made the bar graph below to show the height of the plant at the end of each week. Which is the most reasonable estimate of the plant’s height at the end of the sixth week? (Lesson 1-8)

![Plant Height Graph]

A. 2 to 3.5 cm  B. 4 to 5.5 cm  C. 6 to 7 cm  D. 8 to 8.5 cm

4. WEAT predicted a 25% chance of snow. WFOR said the chance was 1 in 4. Myweather.com showed the chance of snow as \( \frac{1}{5} \), and Allweather.com listed the chance as 0.3. Which forecast predicted the greatest chance of snow? (Lesson 2-7)

A. WEAT  B. WFOR  C. Myweather.com  D. Allweather.com

5. Amber owns a business that transfers photos to CD-ROMs. She charges her customers $24.95 for each CD-ROM. Her expenses include $575 for equipment and $0.80 for each blank CD-ROM. Which of these equations could be used to calculate her profit \( p \) for creating \( n \) CD-ROMs? (Lesson 3-1)

A. \( p = (24.95 - 0.8)n - 575 \)  B. \( p = (24.95 + 0.8)n + 575 \)  C. \( p = 24.95n - 574.2 \)  D. \( p = 24.95n + 575 \)

6. Which of the following equations has the same solution as \( 8(x + 2) = 12 \)? (Lesson 3-4)

A. \( 8x + 2 = 12 \)  B. \( x + 2 = 4 \)  C. \( 8x = 10 \)  D. \( 2x + 4 = 3 \)

7. Eduardo is buying pizza toppings for a birthday party. His recipe uses 8 ounces of shredded cheese for 6 servings. How many ounces of cheese are needed for 27 servings? (Lesson 3-6)

A. 27  B. 32  C. 36  D. 162

8. The sum of \( x \) and \( \frac{1}{y} \) is 0, and \( y \) does not equal 0. Which of the following is true? (Lesson 3-8)

A. \( x = -y \)  B. \( \frac{x}{y} = 0 \)  C. \( x = 1 - y \)  D. \( x = -\frac{1}{y} \)

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**Test-Taking Tip**

Questions 2, 6, 8 Always read every answer choice, particularly in questions that ask, “Which of the following is true?”

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Chapter 3  Solving Linear Equations
Part 2  Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

9. Let \( x = 2 \) and \( y = -3 \). Find the value of \( \frac{x(y + 5)}{4} \). (Lesson 1-2)

10. Use the formula \( F = \frac{9}{5} C + 32 \) to convert temperatures from Celsius (C) to Fahrenheit (F). If it is \(-5^\circ\) Celsius, what is the temperature in degrees Fahrenheit? (Lesson 2-3)

11. Darnell keeps his cotton socks folded in pairs in his drawer. Five pairs are black, 2 pairs are navy, and 1 pair is brown. In the dark, he pulls out one pair at random. What are the odds that it is black? (Lesson 2-6)

12. The sum of the ages of the Kruger sisters is 39. Their ages can be represented as three consecutive integers. What is the age of the middle sister? (Lesson 3-4)

13. On a car trip, Tyson drove 65 miles more than half the number of miles Pete drove. Together they drove 500 miles. How many miles did Tyson drive? (Lesson 3-4)

14. Solve \( 7(x + 2) + 4(2x - 3) = 47 \) for \( x \). (Lesson 3-5)

15. A bookshop sells used hardcover books with a 45% discount. The price of a book was $22.95 when it was new. What is the discounted price for that book? (Lesson 3-7)

Part 3  Quantitative Comparison

Compare the quantity in Column A and the quantity in Column B. Then determine whether:

A  the quantity in Column A is greater,
B  the quantity in Column B is greater,
C  the two quantities are equal, or
D  the relationship cannot be determined from the information given.

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>16. (</td>
<td>a</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>17. solution of ( 3x + 7 = 10 )</td>
<td>solution of ( 4y - 2 = 6 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>18. the percent of increase from $75 to $100</td>
<td>the percent of increase from $150 to $200</td>
</tr>
</tbody>
</table>

Part 4  Open Ended

Record your answers on a sheet of paper.

19. Kirby’s pickup truck travels at a rate of 6 miles every 10 minutes. Nola’s SUV travels at a rate of 15 miles every 25 minutes. The speed limit on this street is 40 mph. Is either vehicle or are both vehicles exceeding the speed limit? Explain. (Lesson 3-6)

20. A chemist has one solution of citric acid that is 20% acid and another solution of citric acid that is 80% acid. She plans to mix these solutions together to make 200 liters of a solution that is 50% acid. (Lesson 3-9)

a. Complete the table to show the liters of 20% and 80% solutions that will be used to make the 50% solution. Use \( x \) to represent the number of liters of the 80% solution that will be used to make the 50% solution.

<table>
<thead>
<tr>
<th>Solution</th>
<th>Liters</th>
</tr>
</thead>
<tbody>
<tr>
<td>20% Solution</td>
<td></td>
</tr>
<tr>
<td>80% Solution</td>
<td>( x )</td>
</tr>
<tr>
<td>50% Solution</td>
<td>200</td>
</tr>
</tbody>
</table>

b. Write an equation that represents the number of liters of acid in the solution.

c. How many liters of the 20% solution and how many of the 80% solution will the chemist need to mix together to make 200 liters of a 50% solution?